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Technical report 97-109

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To cite this work, please refer to the published version:

T. Van Gestel, K. De Cock, R. Jans, B. De Schutter, Z. Degraeve, and B. De Moor, "Discrete stochastic modelling of ATM-traffic with circulant transition matrices," *Mathematical Theory of Networks and Systems* (Proceedings of the MTNS-98 Symposium, held in Padova, Italy, July 1998) (A. Beghi, L. Finesso, and G. Picci, eds.), Padova, Italy: Il Poligrafo, pp. 891–894, 1998.

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Discrete stochastic modelling of ATM-traffic with circulant transition matrices

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***Abstract*— In this paper a new approach to the modelling of ATM-traffic is proposed. The traffic is measured and characterised by its first and second order statistic moments. A Markov Modulated Poisson Process (MMPP) is used to capture the information in these two stochastic moments. Instead of a general MMPP, a *circulant MMPP* is used to reduce the computational cost. A circulant MMPP (CMMPP) is an MMPP with a circulant transition matrix. The main advantages of this approach are that the eigenvalue decomposition is a Fast Fourier Transform and that the optimisation towards the two stochastic moments is decoupled. Based on these properties, a fast time domain identification algorithm is developed.**

I. Introduction

Asynchronous Transfer Mode (ATM) is a protocol for packet switched broadband ISDN networks. Its main characteristic is that it combines the advantages of the classic circuit mode and packet mode traffic. Therefore ATM uses the principle of *statistical multiplexing*, which is very efficient for variable bit rate applications: it is assumed that not every user uses his maximal assigned bit rate, so that more clients can be allocated to a channel than can be processed when every user consumes his maximum bit rate, see e.g. [1] for details.

Buffers are placed in the nodes of the ATM-network to absorb the largest part of the fluctuations in the traffic. Figure 1 gives a schematic representation of a server with finite buffer length. The aggregated arrival pattern $\{a_k\}_{k=1}^{\infty}$ is the sum of the arrival patterns of the individual clients. Each client demands a certain negotiated level of *Quality of Service* (QoS). The following question is important: given the currently processed traffic, is it possible to allocate a new client to the channel so that the new client obtains his level of QoS, without loss of the previously negotiated QoS of the other clients? The new client will certainly increase the load on the server and the QoS will certainly decrease. The amount of this decline is calculated with

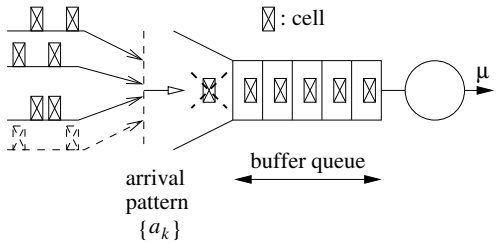

Fig. 1. Schematic representation of a server with finite buffer length. The aggregated traffic $\{a_k\}$ is composed of the traffic of the individual clients. Most of the fluctuations are absorbed in the buffer. No cells are lost as long as the buffer size exceeds the length of the queue. This length and the losses are calculated by queueing analysis and depend on the traffic and the server rate μ.

queueing theory. This could be done by simulation, but this approach is rejected because of the computational requirements. To obtain a swift Connection Admission Control, a better approach is to use a *mathematical model* of the aggregated arrival pattern and use this model in queueing analysis. In this paper a new time domain approach is proposed for the identification of such a mathematical model.

II. Mathematical background

A. Model choice

The main purpose of the mathematical model is to increase the speed of the queueing analysis. Therefore the model only needs to capture the properties important for the queueing analysis. In [2], [3] it is illustrated that the most important features of the traffic are the first and second order statistic moments (probability distribution function and autocorrelation). Moreover the lower frequencies of the arrival pattern affect the queueing analysis the most, which is in fact easy to comprehend.

Therefore *Markov Modulated Poisson Processes* (MMPP) are chosen as model class, since they are easily used in queueing analysis and since it has the flexibility to capture both statistic moments.

A general MMPP is characterised by its transition matrix $P \in \mathbb{P}^{N\times N}$ and its Poisson parameter vector λ, where $\mathbb{P}^{N\times N}$ is the set of all stochastic matrices of dimension N and $\lambda_i \geq 0$ is the Poisson parameter of the Poisson process associated with state i of the Markov chain. Let $s_i(k)$ denote the probability that the Markov chain is in state i at time k, then $s(k+1) = s(k)\cdot P$. In steady state, the probabilities of the different state will not change any more. This state probability is given by $s(\infty) \equiv \pi$, such that $\pi = \pi \cdot P$. The vector π is the left eigenvector of P associated with the eigenvalue 1.

Figure 2 represents an MMPP of order 4. It consists of a stochastic transition matrix P and a vector π with the Poisson parameters. P describes the transition probabilities between the 4 states. The Poisson parameter λ_i of the Poisson process characterises the number of emitted cells when the Markov chain is in that state i.

B. Properties

An MMPP can also be completely described by its statistic moments. Since only the first two moments are important, only these moments of the Markov chain are given here. The probability ($f(x) = Pr\{a_k = x\}$)) and cumulative ($F(x) = Pr\{a_k \leq x\}$) distribution functions

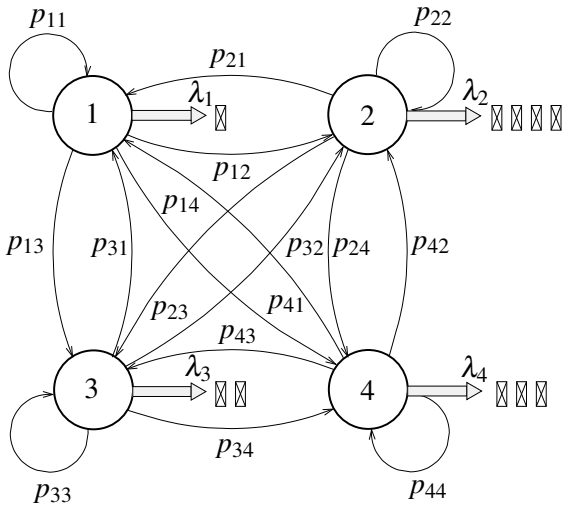

Fig. 2. MMPP of order 4: when the Markov chain is in e.g. state 3, the number of ATM-cells is described by the probability or cumulative distribution function of the associated Poisson process of λ_3, i.e. f_{λ_3} or F_{λ_3}.

are ($x \in \mathbb{N}$):

$$f(x) = \sum_{i=1}^{N} \pi_i f_{\lambda_i}(x) = \sum_{i=1}^{N} \pi_i e^{-\lambda_i} \frac{\lambda_i^x}{x!} \tag{1}$$

$$F(x) = \sum_{i=1}^{N} \pi_i F_{\lambda_i}(x) = \sum_{i=1}^{N} \pi_i e^{-\lambda_i} \sum_{j=0}^{x} \frac{\lambda_i^j}{j!} \quad . \tag{2}$$

The autocorrelation of an MMPP is[1]:

$$R(0) = \pi\Lambda_d e + \pi\Lambda_d^2 e = \overline{\lambda^2} + \overline{\lambda} \tag{3}$$

$$R(n) = \pi\Lambda_d P^n \Lambda_d e \quad (n > 0) \tag{4}$$

$$R(\infty) = \overline{\lambda}^2 \quad . \tag{5}$$

C. The inverse eigenvalue problem

The transition matrix P is a stochastic matrix, which means that $\sum_{i=1}^{N} p_{ij} = 1$, $\forall j$ and that each element $p_{ij} \geq 0$. These restrictions implicate that there are sets of eigenvalues which cannot belong to a stochastic matrix. The theorem of Karpelevič formulates necessary conditions, while sufficient conditions are described in [4] for sets of real eigenvalues. Therefore it is not straightforward to impose an eigenvalue set, which describes the dynamics of the arrival pattern $\{a_k\}$, on an MMPP.

To avoid the inverse eigenvalue problem, a good approach is to start from an MMPP with its restrictions and adapt it such that it resembles the given autocorrelation and distribution function, cf. [5].

[1] $\overline{x}$ denotes the weighted average of x, while e is a column vector of length N, containing all 1's and the index $_d$ in Λ_d stands for a diagonal matrix with the elements of λ on the main diagonal.

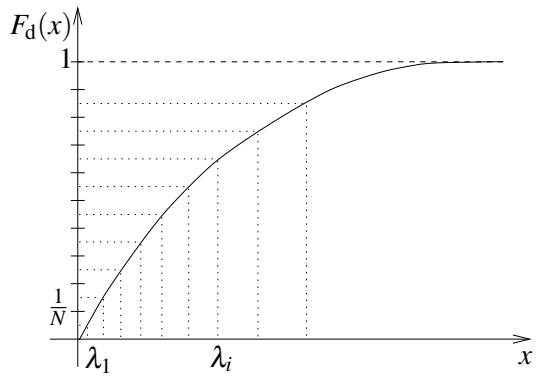

Fig. 3. Graphical representation of the identification procedure of the first order statistics. The cumulative distribution function of the data F_{d} is approximated by a staircase function of Poisson processes with weight $\frac{1}{N}$.

D. Circulant matrices

Analysis of the above equations shows that the numerical complexity of the computations is very high: the calculation of equation (4) requires $\mathscr{O}(N^3)$ operations, while the computation of the equations (1) and (2) requires large computation times because of the calculation of the Poisson distributions, see [6][2] for a comprehensive discussion. Hence, this scheme is not appropriate in an optimisation procedure.

The more restricted model class of *circulant* stochastic matrices [7] is chosen to simplify the computational load. Their eigenvalue decomposition can be written as a Fast Fourier Transform[3] (`fft`): $P \equiv F^* \Gamma_{\mathrm{d}} F$, where $(F)_{ij} = \frac{1}{\sqrt{N}} \exp(-\frac{1}{N} 2\pi\sqrt{-1}(i-1)(j-1))$. This simplifies equation (4) into:

$$R(n) = \frac{1}{N^2} \mathtt{fft}(\lambda)^* \Gamma_{\mathrm{d}}^n \mathtt{fft}(\lambda) \ . \tag{6}$$

III. Identification procedure

The identification procedure consists of two consecutive steps: 1. Identification of the first order statistics: identification of the Poisson parameters; 2. Identification of the second order statistics: firstly the autocorrelation is identified by a linear time-invariant stochastic system, secondly the poles are allocated to the amplitudes of the poles of the linear time-invariant stochastic process and thirdly the poles of the CMMPP are identified.

[2]This report is available by anonymous ftp from `ftp.esat.kuleuven.ac.be` in the directory `pub/SISTA/decock/reports/97-90.ps.gz`

[3]X^* denotes the complex conjugate of X.

A. First order statistics

Circulant stochastic matrices are a subclass of double stochastic matrices and as a consequence the distribution function only depends on the size of the transition matrix ($\pi_i = \frac{1}{N}$) and the Poisson parameters λ_i. The Poisson parameters $\{\lambda_i\}_{i=1}^{N-2}$ are identified by approximating the cumulative distribution function by a (quasi) staircase function, as pointed out in figure 3. The order N is chosen in the range 64 to 128.

Two λ_i's are left over since from equations (3) and (5) also $R(0)$ and $R(\infty)$ depend only on the model order N and the Poisson parameter vector λ. This quadratic equation guarantees that the CMMPP is still consistent with both beginning and end of the autocorrelation and has the additional advantage that also the *mean* of the distribution function is correctly estimated, which is a very important feature in the queueing analysis.

B. Second order statistics

The autocorrelation describes the dynamics of the system. A slowly decaying autocorrelation means highly correlated traffic and increases the load on the buffer.

1) Stochastic system identification: Decomposition of the autocorrelation $R(n)$ in a sum of exponentials is a very compact representation and therefore very well suited in an iterative identification process. The decomposition is done by *stochastic subspace identification* [8], [9] and results in ($N_{\mathrm{s}} \ll N$):

$$R(n) = A_0 + \sum_{i=1}^{N_{\mathrm{s}}} A_i b_i^n \ , \tag{7}$$

where $A_0 = R(\infty)$, because of the stability of the system.

Comparison of equation (4) with equation (7) leads to the following identity:

$$\frac{1}{N^2} \mathtt{fft}(\lambda)^* \Gamma_{\mathrm{d}}^n \mathtt{fft}(\lambda) \equiv A_0 + \sum_{i=1}^{N_{\mathrm{s}}} A_i b_i^n \ . \tag{8}$$

This means that both the (aggregated) amplitudes and the roots of both systems must be the same. The matching is done by solving two problems: an integer programming problem and an optimisation problem. A short description of these problems is given in the following two paragraphs, see [10][4] for a more detailed description.

[4]This report is available by anonymous ftp from `ftp.esat.kuleuven.ac.be` in the directory `pub/SISTA/vangestel/reports/97-108.ps.gz`

2) Integer programming problem: The first problem consists of finding disjunct subsets from the set $C = \{\frac{1}{N^2}|\texttt{fft}(\lambda)_i|^2 : i = 1 \ldots N\}$ so that their sums equal $\{A_i\}$.

The problem of finding a subset from a given set of numbers C with the sum of the numbers in this subset as close as possible to a given number $G = A_i$ can be stated as an integer programming problem. In fact the problem can be formulated as two 0/1 integer knapsack problems: one (P1) for giving the closest approximation which is lower than G and the other (P2) for finding the closest approximation which is larger than G. Using the branch and bound algorithm of Fayard and Plateau [11], the problem can be solved in about 1 second for a typical set of 64 numbers[5].

The knapsack solver is used in an iterative process. First the problem for the largest given number $G = A_i$ is solved. The subset of the current solution is removed from the initial set $C = \{\frac{1}{N^2}|\texttt{fft}(\lambda)_i|^2 : i = 1 \ldots N\}$. Then the knapsack solver is applied to the second largest A_i, now with the reduced set. Although it is obvious that one obtains a sub-optimal solution, this approach is much faster than the optimal solution and also puts the largest weight on the most important exponentials.

3) Optimisation problem: The problem of finding a circulant transition matrix such that the poles γ_i approximate the corresponding poles b_i of the selected amplitudes in the diagonal form of equation (6), is reformulated as a unconstrained optimisation problem by using the parametrisation described in [5]. The sum of the squares of the differences between the given poles and real poles is minimised, using the function `leastsq` in the Matlab® optimisation toolbox, with the Levenberg-Marquardt option.

C. Example

As an example a third order (general) MMPP is identified with a CMMPP of order 64. Data were produced by simulating the original MMPP. A set $\{a_k\}$ of 100 000 points was generated, the latter half was used for the identification procedure to avoid the influence of the choice of the initial state. A third order original model was used since it is illustrated in [5], [6] that higher order models do not necessarily have more complex dynamics, only the first order statistics are influenced. This is also the reason for the apparently large order of the identified model.

Using an HP 9000 Model 712/80 workstation with 64 MB internal memory and clock speed 80 MHz, all computations (except the knapsack solver) took about 60 seconds, starting from raw data. The cumulative distribution function and autocorrelation of the original model (full line) and of the identified CMMPP (dashed line) are given in figures 4 and 5.

[5]A Pentium 66 MHz was used to do the calculation.

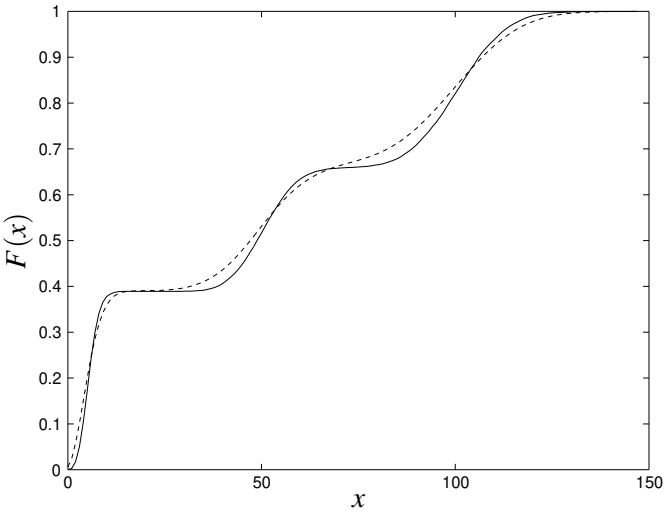

Fig. 4. Comparison of the cumulative distribution function of the original MMPP (full line) and the identified (circulant) MMPP (dashed line).

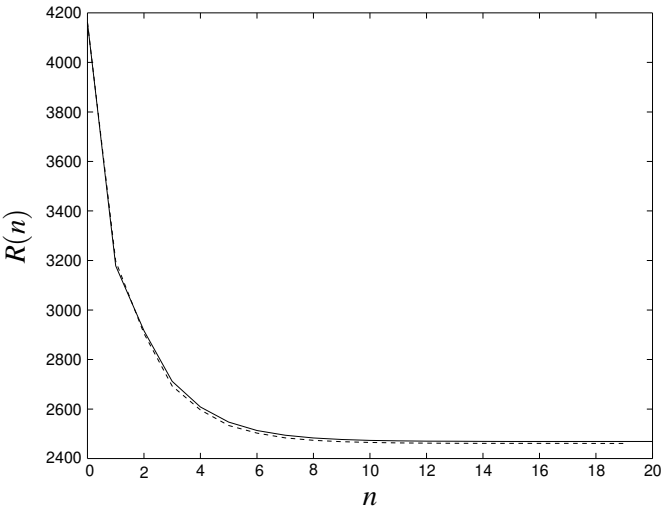

Fig. 5. Comparison of the autocorrelation of the original MMPP (full line) and the identified (circulant) MMPP (dashed line).

One notices that the autocorrelations match very well and that the distribution function is quite well approximated. The error is mainly due to identified Poisson parameters in between the distinct Poisson parameters of the original MMPP.

IV. Conclusions

In this paper a fast time domain approach was presented to identify a circulant Markov Modulated

Poisson Process (MMPP), based on the autocorrelation and the cumulative distribution function. To avoid the inverse eigenvalue problem, optimisation was used. Models with low computational cost were searched for. The use of a circulant MMPP reduces the computational cost and decouples the matching of the first and second order statistic moment.

To further reduce the number of calculations, a quasi staircase approximation of the cumulative distribution function was used, with attention to the beginning and end of the autocorrelation and also the mean of the distribution. The autocorrelation was approximated by subspace-identification as a sum of exponentials, the poles were allocated by 0/1 knapsack solvers and identified by unconstrained optimisation.

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Acknowledgements

This research was done at the K.U.Leuven, Dept. of Electrical Engineering (ESAT), SISTA, Kard. Mercierlaan 94, B-3001 Leuven, Belgium, tel. 32/16/321709, fax. 32/16/321970. T. Van Gestel is a Research Assistant with the Fund for Scientific Research-Flanders (FWO-Vlaanderen), K. De Cock is a Research Assistant with the IWT (Flemish Institute for Scientific and Technological Research in Industry), R. Jans is a Research Assistant with the K.U.Leuven at the Dept. of Applied Economics, B. De Schutter is a Senior Research Assistant with the FWO-Vlaanderen, Z. Degraeve is Associate Professor of management science at the K.U.Leuven Dept. of Applied Economics and B. De Moor is a Senior Research Associate with the FWO-Vlaanderen. This work was supported by the **Flemish Government** (BOF (GOA-MIPS), AWI (Bil. Int. Coll.), FWO (projects, grants, res. comm. (ICCoS)), IWT (IWT-V CST (CVT), ITA (ISIS), EUREKA (Sinopsys))), the **Belgian Federal Government** (IUAP IV-02, IUAP IMechS), the **European Commission** (HCM (Simonet), TMR (Alapedes), ACTS (Aspect), SCIENCE (ERNSI)).