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A Single-Level Rule-Based Model Predictive Control Approach for Energy Management of Grid-Connected Microgrids

Tomás Pippia, Joris Sijs, and Bart De Schutter, *Fellow, IEEE*

Abstract—A single-level rule-based Model Predictive Control (MPC) scheme is presented for optimizing the energy management of a grid-connected microgrid composed of local production units, renewable energy sources, local loads, and several types of energy storage systems. The single-level controller uses two different models that yield different descriptions of the microgrid and that use different sampling times. The model with a smaller sampling time provides a more detailed description of the microgrid, in order to keep track of the fast dynamics, while the model with a higher sampling time provides a less detailed description and is used for making long-term predictions when it is not needed anymore to track the fast dynamics. Moreover, we propose a novel rule-based MPC method that assigns the value to the binary decision variables in the hybrid microgrid model, e.g. ON or OFF status of generators, charging or discharging mode of energy storage systems, through if-then-else rules, which rely on the price of electricity and the local net imbalance. The standard method of applying MPC to a hybrid model results in a Mixed Integer Linear Programming (MILP) problem. Our proposed rule-based method is able to convert the standard MILP problem into a linear one. We compare our approach through simulations to the MILP approach and we show that our method yields almost no loss in performance while providing a significant reduction in the computation time.

I. INTRODUCTION

Energy transition from fossil fuels to renewable energy sources is taking place in many countries, with the goal to improve the sustainability and to decrease the environmental impact of energy production while trying to keep the same services that are provided with the traditional power grid [1]. Indeed, the integration of renewable energy sources poses many challenges, especially for what concerns their intermittent nature [2], [3]. New control actions are being studied in order to facilitate the inclusion of renewable energy sources in the main power grid. Moreover, in recent years, the concept of microgrids has been extensively considered in the scientific literature within the framework of smart grids; see [2]–[5] and the references therein about smart grids and microgrids, and [6]–[17] for recent microgrid-related work. Microgrids are small size electrical grids that include elements such as local production units, local loads, and local energy storage systems. There are many benefits related to the adoption of the concept of microgrids in the presence of renewable energy sources [2]. One of them is related to the fact that the energy produced

locally is also used locally, thus avoiding transportation costs and a decrease in efficiency.

In order to optimize the power flows within the microgrid, a control strategy must be implemented and recently some Model Predictive Control (MPC) schemes have been presented [10]–[17]. The recent work [10] discusses a two-layer energy management system in which the upper layer minimizes the total operational costs and the lower layer tries to mitigate the fluctuations induced by the forecast errors, by using several energy storage systems. The authors of [11] present a procedure for modeling the different components of the microgrid and they apply an MPC algorithm that uses an economical cost function. This work is extended in [12] to a stochastic MPC approach, considering a stochastic controller that uses forecasts of loads and renewable energy sources. A stochastic MPC approach is used also in [13], which presents a hierarchical controller structure, in which the upper level solves an off-line open-loop optimal control problem and the lower level, with knowledge about the stochastic processes within the microgrid, takes care of tracking the solution provided by the upper level. The approach considered in [16] involves a two-level hierarchical MPC controller, where the upper level solves on a long time scale the unit commitment problem, i.e. the problem of deciding whether to turn off or on the local generators, and the lower level solves on a smaller time scale the economic dispatch problem, i.e. the problem of choosing the optimal power flows, once the unit commitment problem is solved. In all the mentioned papers, the overall MPC optimization problem is cast as a Mixed Integer Linear Programming (MILP) problem. Although nowadays there are some efficient solvers to solve this kind of programming problems, e.g. Gurobi [18] or CPLEX [19], the overall worst case complexity of MILP problems is considered to be exponential on the number of optimization variables [20]. Combining this with the fact that usually in microgrid operation optimization the prediction horizon is quite long, i.e. 24h, the overall computational complexity of the MPC-MILP problem can be too high.

Many works have been proposed to solve MILP problems in an efficient way [21]–[23]. In particular, decomposed methods [24], [25] split a specific problem into a “master” problem and one or more “slave” problems. In [24], [25], a set of so called complicating variables, which are the main source of complexity in the problem, are identified in the integer program. The non-complicating variables are projected out of the integer program. The master problem then seeks for a solution to the new integer program, while the slave problem either determines that the master problem is feasible for the original integer program or produces a constraint that it violates. The resulting “Bender cuts” are then added to the original master

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problem. Another approach is the so called “branch-and-price” method [26], [27], where instead of adding new constraints to the (primal) master problem, a separation between feasible and infeasible solutions for the dual of the master problem is used to add new constraints to the dual problem. This approach was first described in [28], together with a decomposition method called Dantzig–Wolfe decomposition. The interested reader is referred to [21]–[23] and the references therein for more details on MILP methods.

Nevertheless, the worst-case computational complexity of MILP problems, like the MPC-MILP problem with a long horizon considered here, remains exponential in the number of integer variables. A possible solution to this issue could be to parameterize the control inputs, such that the parameters of a parameterized control law, and not the control inputs, are optimized [14], [29]–[31]. However, in the case of a model with both binary and continuous variables, this could be challenging, because it would be necessary to parameterize both kinds of variables, and if the parameterized law is non-linear, there could be no improvements in the computational complexity. In order to obtain computational savings with a small loss of performance, it is a better approach to go from an MILP problem to a linear one. Moreover, instead of parameterizing the control variables, another approach could be to assign values to the binary decision variables using if-then-else rules, while still optimizing the continuous variables. In this way, we have a combination of a rule-based controller with an MPC one.

In this article, we propose a novel rule-based MPC controller that assigns the value to the binary decision variables in the hybrid model of a grid-connected microgrid. Our proposed approach solves both the unit commitment problem and the problem of choosing in which mode of operation the batteries and the power exchange with the main grid are set through a set of if-then-else rules. The rules we propose are based on an economic consideration and the values of the different binary variables are assigned based on external variables such as the price of electricity and the local net imbalance. Our approach is able to achieve almost the same performance as the standard controllers available in the literature, while providing a great improvement in the computational time of the control optimization problem. Moreover, we also propose a new control scheme. Unlike previous works, where a hierarchical control scheme for microgrids was presented [11]–[13], [16], we present a single-level controller that uses two different models of the microgrid under control. The two models have two different sampling times and the model with the smallest sampling time is used in the MPC algorithm for predictions that are close in time to the time step at which the MPC optimization problem is solved. On the other hand, the model with the larger sampling time is used for predictions that are far in time with respect to the sampling time at which the MPC optimization problem is solved. In other words, we fix a time threshold, before which the ‘fast’ model is used and after which the ‘slow’ model is used. The idea is that when we compute the control action for the current sampling time, we have to consider the evolution of the system right after we apply our control action, while after some time steps we

can consider a more granular model, since all the information related to prices, local renewable energy production, and loads is also more granular. Moreover, some storage systems, e.g. ultracapacitors, could have, by their nature, a limited amount of time during which they can hold an electric charge, which means that these devices should not be considered in the prediction for time instants far in time, i.e. after which the charge stored in them has vanished. With this approach, we are able to control the microgrid, in the tertiary control level, by solving only one optimization problem that uses information about the future evolution of the microgrid. In other words, instead of using a hierarchical control scheme within the tertiary control layer that has to consider two different control problems, we combine them into only one optimization problem while still using all the available information related to the microgrid. By using this approach, we provide a simpler MPC controller structure and avoid possible implementation problems, e.g. delays, overhead, arising from the extra control layer. Moreover, hierarchical approaches within the tertiary control layer usually have a large time step T_s , i.e. at least 1h, for the upper layer. Therefore, the predictions are updated only once every T_s time units. With our method we can instead update the predictions much more often, i.e. once every T_f time units, where T_f is the time step of the ‘fast’ model. This allows to have a faster reaction to possible changes in the predictions of loads and renewables, e.g. a sudden increase in the load. The increase in the computational complexity, related to the fact that at each time step we solve an online optimal control problem with a large prediction horizon, is reduced both with our proposed if-then-else rules and through the use of the two models, since one of them has a large sampling time.

We would like to stress that although there exist some efficient commercial solvers for solving MILP problems, e.g. [18], [19], their computational complexity is still exponential in the worst case, and the problem remains an NP-hard one [20]–[22], [32]. While our rule-based approach does not have the theoretical guarantees of MILP methods, it provides a good compromise between computational complexity and performance and stands out as an alternative to MILP methods. Indeed, with our method we get rid of the binary variables and therefore the computational complexity becomes polynomial, hence the computation time is greatly reduced. In applications in which the solving time of the optimization problem is not required to be small, one can use the MILP approach and obtain the best achievable performance. On the other hand, when time is crucial, hardware is limited, or when the size of the problem is too large to be solved efficiently in a limited amount of time, it is advisable to look for alternative solutions other than the MILP approach, and our method fits in this category.

The contribution of this article is therefore twofold:

- we propose a new control scheme that, in contrast to hierarchical control, merges the two control levels into one single level, by using two models with different sampling times;
- we present a novel set of if-then-else rules for assigning the value to the binary decision variables involved in the

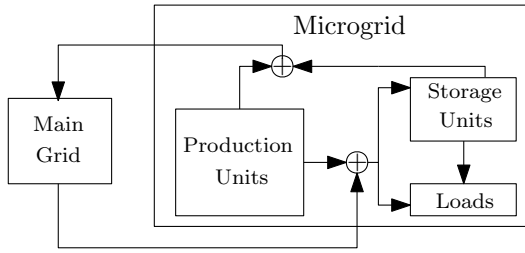


Fig. 1. Microgrid scheme considered in this article. Arrows represent power flows. The microgrid is connected to the main grid.

optimization procedure of the microgrid operation.

This approach is able to provide almost the same performance as standard controllers but with a significant decrease in the computational complexity, as will be shown through simulations.

The outline of the article is as follows. In Section II, we present the model of the microgrid under control and our proposed single-level controller. In Section III we present our novel rule-based MPC method. Lastly, in Section IV we show the application of our proposed method in simulations and we draw conclusions and remarks for future work in Section V.

Notation: We denote vectors and matrices with bold style, e.g. \mathbf{x} , \mathbf{P} , and scalars with non-bold text style, e.g. x , P . Moreover, the operator $\lceil \cdot \rceil$ represents the ceiling operator, i.e. $\lceil x \rceil$ is equal to the closest integer $x_{\text{int}} \geq x$.

II. MODEL DESCRIPTION

In this article, we consider a microgrid that includes several elements, as shown in Figure 1. These elements are storage units (i.e. batteries and ultracapacitors), sources (i.e. renewable sources and local dispatchable units), a bidirectional connection to the main grid (i.e. energy can be bought or sold), and uncontrollable loads. Moreover, we consider the operational economical costs of the microgrid, i.e. the costs for producing electricity locally and buying electricity from the main grid. The goal in microgrid operation optimization is to minimize the economical costs, optimally choosing the power flows within the microgrid and the exchange of power with the main grid.

In this section, we first describe the model of the system under control, then we introduce the novel single-level controller proposed in this article, and lastly we introduce the model constraints, which are both related to the power exchange and the modeling framework.

A. Microgrid Description

The microgrid model that we consider is similar to the one presented in [11], with some modifications, in order to better adapt it to our case.

a) Dynamics of the energy storage systems: The dynamics of the Energy Storage Systems (ESSs) are expressed with the simplified formulation presented in [33] with respect to [11], i.e.

$$x_{\text{st}}(h+1) = \begin{cases} x_{\text{st}}(h) + \frac{T_s}{\eta_{\text{d,st}}} P_{\text{st}}(h), & P_{\text{st}}(h) < 0 \\ x_{\text{st}}(h) + T_s \eta_{\text{c,st}} P_{\text{st}}(h), & P_{\text{st}}(h) \geq 0 \end{cases}, \quad (1)$$

where $x_{\text{st}}(h)$ indicates the level of energy stored at the ESS at time step h , $\eta_{\text{c,st}}$ and $\eta_{\text{d,st}}$ are the charging and discharging efficiencies, respectively, $P_{\text{st}}(h)$ is the power exchanged with the ESS at time step h , and T_s is the sampling interval of the discrete-time system. At each time step h , the ESS can only be in one of the two modes, i.e. either in the charging or in the discharging mode. In order to model this hybrid behavior, we follow the modeling approach of [11] using a Mixed Logical Dynamical (MLD) model [34] to model the two different modes of the batteries. The boolean variable $\delta_{\text{st}}(h)$ indicates whether the ESS is in the charging or discharging mode at time step h , i.e. $\delta_{\text{st}}(h) = 1 \iff P_{\text{st}}(h) \geq 0$, and $\delta_{\text{st}}(h) = 0 \iff P_{\text{st}}(h) < 0$. Then we define a new auxiliary variable z_{st} as $z_{\text{st}}(h) = \delta_{\text{st}}(h)P_{\text{st}}(h)$ and we can write (1) more compactly in linear form as

$$x_{\text{st}}(h+1) = x_{\text{st}}(h) + T_s \left(\eta_{\text{c,st}} - \frac{1}{\eta_{\text{d,st}}} \right) z_{\text{st}}(h) + \frac{T_s}{\eta_{\text{d,st}}} P_{\text{st}}(h). \quad (2)$$

In this article we consider two different ESSs: an ultracapacitor used for fast response and a battery for storing larger amounts of energy for a longer time span. Note that for simplicity of expression, the number of storage devices here is kept limited but our approach can also be applied to systems with a higher number of ESSs. Moreover, our approach can be used with any kind of ESSs and it is not specific only for batteries and ultracapacitors.

b) Loads: We consider critical loads, i.e. loads that must be satisfied at all times. We denote by $P_{\text{load}}(h)$ the total power required by the loads at time step h . It is assumed that the information on the values of P_{load} is available, either through available information in the microgrid or through forecasting methods.

c) Generators: We consider two different kinds of generators, i.e. dispatchable generators, whose output power can be controlled with a certain degree of freedom, and non-dispatchable generators, whose output power cannot be controlled. Renewable sources are considered as non-dispatchable generators and their output is considered as a known disturbance, since it is a signal that cannot be controlled. We denote by P_{res} the variable representing the power produced by renewable energy sources and \mathbf{P}_{dis} the vector representing the power produced by dispatchable generators, where $\mathbf{P}_{\text{dis}} = [P_1^{\text{dis}}, \dots, P_{N_{\text{gen}}}^{\text{dis}}]^{\top}$ and P_i^{dis} indicates the power produced by dispatchable unit i , $i \in \{1, \dots, N_{\text{gen}}\}$, with N_{gen} denoting the total number of generators. Moreover, we use a variable $\delta_i^{\text{on}}(h)$ to indicate whether dispatchable generator i is active at time step h , i.e. $\delta_i^{\text{on}}(h) = 1$, or not, i.e. $\delta_i^{\text{on}}(h) = 0$.

d) Energy prices: We consider time-varying electricity prices, such that prices for purchase and sale of electricity are different. We denote with $c_{\text{sale}}(h)$ and $c_{\text{pur}}(h)$ the price for selling and purchasing electricity to and from the main grid, respectively. We also consider a time-varying tariff $c_{\text{prod}}(h)$ for producing electricity with the local dispatchable production units.

e) *Main grid*: The microgrid is connected to the main grid and power can flow bidirectionally. Following once again the MLD model of [11], we model the connection with the grid using a binary variable $\delta_{\text{grid}}(h)$ that indicates the direction of the power flow at time step h , i.e. whether energy is being bought from the main grid or sold to it. Denoting by P_{grid} the power exchanged with the main grid, we have

$$\begin{cases} \delta_{\text{grid}}(h) = 0 \iff P_{\text{grid}}(h) < 0, \text{ (exporting case)} \\ \delta_{\text{grid}}(h) = 1 \iff P_{\text{grid}}(h) \geq 0, \text{ (importing case)} \end{cases} \quad (3)$$

We can then define an auxiliary variable C_{grid} as

$$\begin{cases} C_{\text{grid}}(h) = c_{\text{sale}}(h)P_{\text{grid}}(h), P_{\text{grid}}(h) < 0, \\ C_{\text{grid}}(h) = c_{\text{pur}}(h)P_{\text{grid}}(h), P_{\text{grid}}(h) \geq 0, \end{cases} \quad (4)$$

As explained in Section II-C, we can link together δ_{grid} and C_{grid} by resorting to a set of linear constraints. The auxiliary variable C_{grid} is used in the cost function, presented in Section II-D.

f) *Dimensioning of the microgrid*: We apply our method to an already dimensioned microgrid, therefore we assume that the parameters related to all the elements of the microgrid are given. However, it is possible to optimize the sizing of the elements of the microgrid by following the procedure illustrated in e.g. [35], [36].

Remark 1: In this article, we consider that the renewable energy source profiles, the load profiles, and the time-varying prices are known in advance. For what concerns the prices, this is not a limiting assumption, since in some cases these are known some time in advance; see e.g. [37], where authors use a *day-ahead* pricing scheme. As regards the loads and the renewable energy sources, some works [11]–[13] have considered a prediction scheme that provides a predicted load or renewable energy signal to the MPC controller. Although in the current article we do not consider a prediction scheme, it can be easily included in the control scheme presented here and predicted signals can be used instead of signals known *a priori*. In conclusion, we do not explicitly include a prediction scheme because our focus is on presenting the if-then-else rules of Section III, and these rules can be achieved with or without a prediction scheme as long as there is available information on the load, renewables, and prices profiles.

Remark 2: In this article we consider a microgrid in a grid-connected mode as done in many other works in the literature, e.g. [8], [10], [11], [13], [15], [16], [33], [38]–[45]; however, in an application in which it is crucial to minimize the power exchanged with the main grid, e.g. in a case in which the utility grid acts as a backup, it is possible to add a penalization term in the cost function to the power exchanged with the main grid.

B. Fast and Slow Model

In this article, we propose a novel method that uses two different microgrid models, namely a ‘fast’ one and a ‘slow’ one. The ‘fast’ model is used for predictions that are close to the current sampling time, while the ‘slow’ one is used for predictions that are farther away in time. The reason

for choosing such a control structure is twofold. Firstly, the ultracapacitor cannot hold electric charge efficiently for a long time [46], i.e. it has a high self-discharge rate; this holds for other similar ESSs with small capacity. Therefore we assume that the ultracapacitor is available only close to the current sampling time; hence it is used only in the ‘fast’ model, when a quick response is needed for providing or absorbing a small amount of energy. Secondly, the available future data on prices, load, and renewables profiles is denser in time steps close to the current one, while it becomes more sparse far from the current time step. The previous discussion implies that the number of state components and input components of the two models are different. Indeed, the ‘slow’ model does not consider the dynamics of the ultracapacitor and only considers the dynamics of the battery.

The advantage of this kind of controller structure is that, by having only one controller, we reduce the complexity of the control architecture and we make the implementation easier, with respect to hierarchical controllers. Moreover, during each optimization procedure the most updated information is used, compared to other approaches in which information is only updated at a higher level after a certain amount of time.

We show in Figure 2 the different sampling times and the time intervals during which each model is used. We denote by T_s^f and T_s^s the sampling interval of the ‘fast’ and ‘slow’ model, respectively, and we denote by h and k the time steps of the ‘fast’ and ‘slow’ model, respectively. Moreover, we suppose that from time step $N_{f,s}$ of the ‘fast’ model we start using the ‘slow’ model for predictions. Therefore, the step $N_{f,s}$ of the ‘fast’ model coincides with time step 0 of the ‘slow’ model. We also assume that $N_{f,s}T_s^f = T_s^s$, i.e. the ‘fast’ model is used for exactly one time step of the ‘slow’ model.

The dynamic equations of the fast model, by using (2), are

$$\mathbf{x}_f(h+1) = \mathbf{x}_f(h) + B_1^f \mathbf{z}_f(h) + B_2^f \mathbf{u}_f(h), \quad (5)$$

where $\mathbf{x}_f(h) = [x_{f,b}(h) \ x_{f,uc}(h)]^\top$, with $x_{f,b}$ and $x_{f,uc}$ being the storage level of the battery and of the ultracapacitor, respectively, \mathbf{z}_f is the auxiliary variable for the ‘fast’ model, and $B_1^f \in \mathbb{R}^{2 \times 2}$, $B_2^f \in \mathbb{R}^{2 \times m_f}$. We define the input vector as $\mathbf{u}_f(h) = [P_{f,b}(h) \ P_{f,uc}(h)]^\top$, $\mathbf{u}_f(h) \in \mathbb{R}^{m_f}$, which represents respectively the power exchanged with the battery and the power exchanged with the ultracapacitor. The ‘slow’ model is defined in a similar way, i.e.

$$x_s(k+1) = x_s(k) + B_1^s z_s(k) + B_2^s u_s(k), \quad (6)$$

where $x_s(k) = x_{s,b}(k)$ is the storage level of the battery, z_s is the auxiliary variable for the ‘slow’ model, and $B_1^s, B_2^s \in \mathbb{R}$. The input vector is defined as $u_s(k) = P_{s,b}(k)$ and it represents the power exchanged with the battery. Note that, as highlighted before, the number of states and inputs is different between the two models.

We consider the power balance constraint in the microgrid,

$$P_{f,b}(h) = \sum_{i=1}^{N_{\text{gen}}} P_i^{\text{dis}}(h) + P_{\text{res}}(h) + P_{\text{grid}}(h) - P_{f,uc}(h) - P_{\text{load}}(h), \quad (7)$$

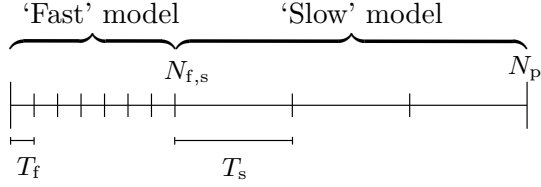


Fig. 2. Scheme adopted in this paper for the time steps of the two different models.

$\forall h \geq 0$, and apply it to (5) to write the expression of the dynamics of the storages as a function of P_{grid} , P_{load} , and P_{p} . Then, by introducing matrices M_u^f , M_w^f and defining $\mathbf{u}^f(h) = M_u^f \bar{\mathbf{u}}_f(h) + M_w^f \mathbf{w}_f(h)$, with $\bar{\mathbf{u}}_f(h) = \begin{bmatrix} \mathbf{P}_{\text{dis}}^\top(h) & P_{\text{grid}}(h) & P_{f,\text{uc}}(h) & (\delta^{\text{on}}(h))^\top \end{bmatrix}^\top$, $\mathbf{w}_f(h) = [P_{\text{load}}(h) \quad P_{\text{res}}(h)]^\top$, we can put together (5) and (7) as

$$\mathbf{x}_f(h+1) = \mathbf{x}_f(h) + B_1^f \mathbf{z}_f(h) + B_2^f (M_u^f \bar{\mathbf{u}}_f(h) + M_w^f \mathbf{w}_f(h)). \quad (8)$$

A similar expression is obtained for (6) by using the version of the ‘slow’ model of (7), where, however, $P_{f,\text{uc}}(h)$ does not appear, i.e.

$$P_{s,\text{b}}(k) = \sum_{i=1}^{N_{\text{gen}}} P_i^{\text{dis}}(k) + P_{\text{res}}(k) + P_{\text{grid}}(k) - P_{\text{load}}(k), \quad (9)$$

$\forall k \geq 0$. Equation (7) is used only for the ‘fast’ model, while (9) is used only for the ‘slow’ model. Similar to what we did before, we introduce matrices M_u^s , M_w^s and define $\mathbf{u}_s(k) = M_u^s \bar{\mathbf{u}}_s(k) + M_w^s \mathbf{w}_s(k)$, with $\bar{\mathbf{u}}_s(k) = \begin{bmatrix} \mathbf{P}_{\text{dis}}^\top(k) & P_{\text{grid}}(k) & (\delta^{\text{on}}(k))^\top \end{bmatrix}^\top$, $\mathbf{w}_s(k) = [P_{\text{load}}(k) \quad P_{\text{res}}(k)]^\top$. Next, we merge (6) and (9) as

$$x_s(k+1) = x_s(k) + B_1^s \mathbf{z}_s(k) + B_2^s (M_u^s \bar{\mathbf{u}}_s(k) + M_w^s \mathbf{w}_s(k)). \quad (10)$$

Since the ‘fast’ and the ‘slow’ model have different input components and state components, it is necessary to define a way to link the two models. As stated before, we assume that the ‘fast’ model is used only until the time instant $T_s^f N_{f,s}$, i.e. time step $N_{f,s}$ of the ‘fast’ model and after that the ‘slow’ model is used. We can then link the two models as

$$x_s(0) = x_{f,\text{b}}(N_{f,s}), \quad (11)$$

which means that we can define a matrix $M_{f,s} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ to link the two models as $x_s(0) = M_{f,s} \mathbf{x}_f(N_{f,s})$.

C. Constraints

In this section, we introduce the constraints related to the models and the power flows in the microgrid. Since we are using an MLD model for the storages and the power exchanged with the main grid, we define the constraints as in [11], [34] by defining matrices E_1, E_2, E_3, E_4 such that we can write the constraints in a compact form. We define two different sets of constraints, one for each model, denoting with a superscript

‘f’ and ‘s’ the constraints for the ‘fast’ and ‘slow’ model, respectively. We can then write compactly the constraints as

$$E_1^f \delta_f(k) + E_2^f \mathbf{z}_f(k) \leq E_3^f \mathbf{u}_f(k) + E_4^f. \quad (12)$$

A similar inequality holds for the ‘slow’ model, where we replace the matrices E_i^f with matrices E_i^s , $i \in \{1, \dots, 4\}$ and we replace the variables of the ‘fast’ model with the ones related to the ‘slow’ model.

We define also constraints on the upper and lower bounds for the states and the inputs, i.e.

$$\underline{P}_{\text{b}} \leq P_{\text{b}}(h) \leq \bar{P}_{\text{b}} \quad (13)$$

$$\underline{P}_{\text{uc}} \leq P_{\text{uc}}(h) \leq \bar{P}_{\text{uc}} \quad (14)$$

$$\underline{P}_{\text{grid}} \leq P_{\text{grid}}(h) \leq \bar{P}_{\text{grid}} \quad (15)$$

$$\delta_i^{\text{on}}(h) \underline{P}_{\text{dis}} \leq P_i^{\text{dis}}(h) \leq \delta_i^{\text{on}}(h) \bar{P}_{\text{dis}} \quad (16)$$

$$\underline{x}_{\text{st}} \leq \mathbf{x}_f(h) \leq \bar{x}_{\text{st}} \quad (17)$$

for $i \in \{1, \dots, N_{\text{gen}}\}$, where $\underline{x}_{\text{st}} = [\underline{x}_{\text{b}} \quad \underline{x}_{\text{uc}}]^\top$, and \underline{x}_{b} , $\underline{x}_{\text{uc}}$ are the lower bounds for the state of charge of the battery and the ultracapacitor, respectively, and $\bar{x}_{\text{st}} = [\bar{x}_{\text{b}} \quad \bar{x}_{\text{uc}}]^\top$, and \bar{x}_{b} , \bar{x}_{uc} are respectively the lower bound for the state of charge of the battery and the ultracapacitor. The constraints (13)-(17) model the physical bounds on, respectively, the power exchanged with the battery, the power exchanged with the ultracapacitor, the power exchanged with the main grid, the power produced by each production unit, and the level of charge of the ESSs. The constraints (13)-(17) are expressed using the sampling time index of the ‘fast’ model, i.e. h , but they are also applied to the variables of the ‘slow’ model as well.

Moreover, we also consider constraints for the generators related to the amount of time they should stay turned on or off. Following [11], we denote by T_{on}^f and T_{off}^f the minimum amount of time during which the generators should be turned on or off, respectively, expressed in number of sampling times of the ‘fast’ model. Therefore, we can introduce the constraints

$$\delta_i^{\text{on}}(h) - \delta_i^{\text{on}}(h-1) \leq \delta(\bar{l}), \text{ from OFF to ON} \quad (18)$$

$$\delta_i^{\text{on}}(h-1) - \delta_i^{\text{on}}(h) \leq 1 - \delta(\bar{l}), \text{ from ON to OFF} \quad (19)$$

for $\bar{l} = 1, \dots, \min\{h + T_{\text{off}}^f - 1, N_{\text{p}}\}$ and $\bar{l} = 1, \dots, \min\{h + T_{\text{on}}^f - 1, N_{\text{p}}\}$. Again, this constraint is written using the sampling time of the ‘fast’ model but it is adapted and applied to the ‘slow’ model too. Notice however that since the two models are used consecutively, it might happen that $h + T_{\text{on}}^f - 1 \geq N_{f,s}$ or $h + T_{\text{off}}^f - 1 \geq N_{f,s}$, which means that the constraint associated to the ‘fast’ model would extend over the time instants of the ‘slow’ model. In that case, we extend constraints to the ‘slow’ model defining a \hat{k} as

$$\hat{k} = \left\lceil \frac{T_s^f (h + T_{\text{on}}^f - 1)}{T_s^s} \right\rceil. \quad (20)$$

We then impose the adapted constraints (18), (19) for the ‘slow’ model until time step \hat{k} . The extension for T_{off}^f is done in a similar way.

D. Cost Function

In the MPC scheme we use a cost function that is a sum of several economic terms. In particular, we consider the simple sum of costs and revenues, i.e. we sum up the costs for producing electricity locally and buying electricity from the main grid and the revenues obtained from selling electricity to the main grid. The resulting cost function is defined as

$$J(\mathbf{P}_p(h), C_{\text{grid}}(h)) = \sum_{j=0}^{N_{f,s}-1} \left(C_{\text{grid}}(h+j) + c_{\text{prod}}(h) \sum_{i=1}^{N_{\text{gen}}} P_i^{\text{dis}}(h+j) \right) + \sum_{l=0}^{N_p-1} \left(C_{\text{grid}}(h+N_{f,s}+l) + c_{\text{prod}}(k) \sum_{i=1}^{N_{\text{gen}}} P_i^{\text{dis}}(h+N_{f,s}+l) \right), \quad (21)$$

where the first summation term corresponds to the ‘fast’ model, from time step h until time step $h + N_{f,s} - 1$, and the second summation term corresponds to the ‘slow’ model, from time step $h + N_{f,s}$ until time step $h + N_p - 1$ (recall Fig. 2).

Note that the total cost depends strongly on the prices c_{sale} , c_{pur} , which are included in the variable C_{grid} , and on the price c_{prod} . This fact will be exploited in the next section in the proposed if-then-else procedure.

III. RULE-BASED MPC CONTROLLER

In this section, we present briefly the MPC framework and our proposed rule-based MPC controller, based on if-then-else rules, for microgrid control. We also discuss some issues related to the constraints that need special attention once the if-then-else rules have been applied. Lastly, we present our proposed single-level controller that uses the two models introduced in Section II-B.

A. Model Predictive Control

MPC is an established control approach that has been extensively studied and successfully applied in many fields in the last forty years [47]–[50]. At each time step, an online optimal control problem is solved, using a model of the system under control for computing predictions of the future states up to a certain prediction horizon N_p . The optimization problem results in a sequence of optimal inputs, but only the first element in the sequence is applied to the system. At the following time step, the system state is sampled and a new optimization problem is solved, shifting the prediction horizon one time step forward. Thanks to this strategy, MPC controllers are able to handle, to a certain extent, uncertainties, model mismatches, and disturbances [47]–[49]. Moreover, since the MPC strategy turns the control problem into an optimization one, constraints on the inputs, states, and outputs can be naturally included into the control problem.

When it comes to applying MPC to hybrid systems e.g. MLD systems, the MPC optimization problem becomes a mixed integer programming problem [50], due to the presence

of integer variables in the hybrid model. Many efficient solvers have been developed to solve MILP problems, e.g. [18], [19]. However, in the worst case scenario, the computational complexity of such problems is considered to be exponential [20], i.e. the complexity increases exponentially with the number of integer variables involved. Therefore, in order to reduce the computation time of the control problem, in this article we propose a combination of a rule-based controller and an MPC one, which is discussed in detail in the next subsections.

B. Structure of the Rule-Based MPC Controller

In our proposed rule-based MPC method, we assign the value of the binary variables in the MLD model while optimizing the continuous variables. In this way, the problem is not an MILP one anymore, since the value of the binary variables is assigned before the optimization takes place. Therefore, computational savings are achieved due to the removal of the binary variables in the optimization problem, which becomes then a linear programming one.

The if-then-else rules are designed in order to avoid losses on the performance, which in this case is of an economic nature. Therefore, the rules are based on economic quantities. Moreover, the rules also consider the local renewable energy production and load profiles, since the decisions taken depend on these two quantities too.

C. Assignment of the Values to the Binary Decision Variables

We will first start by analyzing which are the causes that lead the controller to take certain actions, i.e. what triggers the assignment of the binary decision variables in the optimization problem.

In the considered system, the economic cost (21) to be optimized depends on two main quantities: the locally produced power P_{dis} and the power exchanged with the main grid P_{grid} . These two inputs are weighted in the cost function by the price of producing energy locally, c_{prod} , and the prices of electricity purchase or sale, c_{pur} and c_{sale} , respectively, according to whether the microgrid is purchasing electricity or selling it. Since the quantities P_{dis} and P_{grid} are two inputs of the system, these inputs directly determine the overall cost. Note that the price c_{prod} and the input P_{dis} appear directly in the cost function (21), while the prices c_{sale} and c_{pur} and the input P_{grid} appear indirectly inside the variable C_{grid} .

Besides these two quantities, the system must satisfy the power balance constraints (7), (9) at all times. Therefore, the decisions that the controller takes are based on the satisfaction of these constraints. Since the cost of producing energy through the renewable energy sources is considered to be zero, because in our cost function (21) we consider only marginal costs and revenues and not fixed ones, in order to minimize the cost the controller will try to satisfy the loads with the renewable energy sources first, and only if this power is not enough, it will either buy power from the main grid or produce it locally through the dispatchable units. If the power produced by the renewable energy sources is higher than the one required by the loads, then the surplus power can be stored

in the battery or it can be sold to the main grid. Moreover, we must also make a distinction between whether the microgrid is able to completely satisfy the local demand or not, i.e. whether it is necessary to acquire power from the main grid or not.

From the previous discussion, it is possible to notice that the actions that the controller takes are based on two main facts. The first one is related to whether the microgrid can satisfy the local demand with the local production units or not and it is mainly related to the *feasibility* of the control action, i.e. it is closely related to constraints (7), (9). The second one is related to the choice of the power source that will satisfy constraints (7), (9) and it deals with the *optimality* of the control action.

We can then determine the values of the binary decision variables by looking at whether the microgrid can locally satisfy the loads, i.e. we check whether $N_{\text{gen}}P_{\text{P}}^{\text{max}} + P_{\text{res}}(h) < P_{\text{load}}(h)$ and whether $P_{\text{res}}(h) > P_{\text{load}}(h)$. Then, we check the relation between the energy prices, i.e. we check whether $c_{\text{prod}}(h) < c_{\text{sale}}(h) \leq c_{\text{pur}}(h)$, $c_{\text{sale}}(h) < c_{\text{prod}}(h) \leq c_{\text{pur}}(h)$, or $c_{\text{sale}}(h) < c_{\text{pur}}(h) \leq c_{\text{prod}}(h)$. Based on this, we have five different cases:

- 1) $P_{\text{res}}(h) \geq P_{\text{load}}(h)$, with $c_{\text{prod}}(h) < c_{\text{sale}}(h) \leq c_{\text{pur}}(h)$. In this case, the renewable energy sources completely satisfy the loads. Since $c_{\text{prod}}(h) < c_{\text{sale}}(h)$, it is also convenient to produce energy and sell it to the main grid. Furthermore, due to the fact that there is a surplus of energy, the battery is allowed to store energy, because it could be useful to store energy for later usage. Therefore, in this case we impose $\delta_{\text{grid}}(h) = 0$, $\delta_i^{\text{on}}(h) = 1$, $i \in \{1, \dots, N_{\text{gen}}\}$, $\delta_{\text{b}}(h) = 1$. In the resulting optimization problem, it will be determined how much energy to sell to the main grid and to store in the batteries, since these two actions are enabled.
- 2) $N_{\text{gen}}\bar{P}_{\text{dis}} + P_{\text{res}}(h) \geq P_{\text{load}}(h)$ and $P_{\text{load}}(h) > P_{\text{res}}(h)$, with $c_{\text{prod}}(h) < c_{\text{sale}}(h) \leq c_{\text{pur}}(h)$, or $c_{\text{sale}}(h) < c_{\text{prod}}(h) \leq c_{\text{pur}}(h)$. In order to satisfy the local loads, a certain amount of energy has to be acquired, since the renewable energy sources do not completely satisfy the loads. Producing energy is cheaper than buying it from the main grid, so the required energy is produced locally. If $c_{\text{prod}}(h) < c_{\text{sale}}(h)$, then extra energy will be produced in order to be sold to the main grid, otherwise only the necessary energy will be produced. The battery is allowed to store energy, as in the previous cases. Therefore, we impose $\delta_{\text{grid}}(h) = 0$, $\delta_i^{\text{on}}(h) = 1$, $i \in \{1, \dots, N_{\text{gen}}\}$, $\delta_{\text{b}}(h) = 1$.
- 3) $N_{\text{gen}}\bar{P}_{\text{dis}} + P_{\text{res}}(h) < P_{\text{load}}(h)$, with $c_{\text{prod}}(h) < c_{\text{sale}}(h) \leq c_{\text{pur}}(h)$, or $c_{\text{sale}}(h) < c_{\text{prod}}(h) \leq c_{\text{pur}}(h)$. The local loads require more energy than the energy that can be locally produced together by the renewable energy sources and the dispatchable units. Therefore, we set the production units to produce energy and we buy the remaining required energy from the main grid. The battery is also allowed to provide the stored energy. In this case, there is no distinction between the cases $c_{\text{prod}}(h) < c_{\text{sale}}(h)$ and $c_{\text{prod}}(h) \geq c_{\text{sale}}(h)$, since in both cases it is necessary to buy energy from the main grid. Therefore, we impose $\delta_{\text{grid}}(h) = 1$, $\delta_i^{\text{on}}(h) = 1$, $i \in \{1, \dots, N_{\text{gen}}\}$, $\delta_{\text{b}}(h) = 1$.

- 4) $P_{\text{res}}(h) \geq P_{\text{load}}(h)$, with $c_{\text{sale}}(h) < c_{\text{prod}}(h) \leq c_{\text{pur}}(h)$. With these conditions, the renewable energy sources completely satisfy the loads. Since $c_{\text{sale}}(h) < c_{\text{prod}}(h)$, the generators are turned off. Due to the surplus of energy, the battery is allowed to store energy and the grid power exchange is set to the sale mode. The battery can store energy in this case since the price for electricity might increase in the following time steps, or the amount of available renewable energy could be smaller. The optimization procedure will decide whether to sell energy, store it, or perform both actions. For this case we impose $\delta_{\text{grid}}(h) = 0$, $\delta_i^{\text{on}}(h) = 0$, $i \in \{1, \dots, N_{\text{gen}}\}$, $\delta_{\text{b}}(h) = 1$.
- 5) $N_{\text{gen}}\bar{P}_{\text{dis}} + P_{\text{res}}(h) \geq P_{\text{load}}(h)$ and $P_{\text{load}}(h) > P_{\text{res}}(h)$, with $c_{\text{sale}}(h) < c_{\text{pur}}(h) \leq c_{\text{prod}}(h)$. As in the previous case, some energy has to be acquired in order to satisfy the local loads. Producing energy is more expensive than buying it from the main grid, thus the required energy is bought from the main grid and the generators are turned off. In order to reduce the cost, the battery is allowed to provide some stored energy. In this case, we impose $\delta_{\text{grid}}(h) = 1$, $\delta_i^{\text{on}}(h) = 0$, $i \in \{1, \dots, N_{\text{gen}}\}$, $\delta_{\text{b}}(h) = 0$.

All the cases are summarized in Table I. For what concerns the ultracapacitor, in all the mentioned cases we opt to impose a mode of operation that is always the same as the battery. Also, consider the two following observations on Table I. The first one is that in order to optimize the economic costs, in cases 1 and 2 of Table I, we could add an extra step and force the local dispatchable units to produce at full capacity, i.e. \bar{P}_{dis} , since that would be the most convenient choice. However, we could run into infeasibility issues, as will be discussed in the next subsection. Therefore, in these two cases we only force the generators to be turned on and, if it is more profitable *and* feasible, the solver will set the production output to its maximum value. The second observation is that in the first column, $P_{\text{load}} \leq P_{\text{res}}$ implies that $N_{\text{gen}}\bar{P}_{\text{dis}} \geq P_{\text{load}} - P_{\text{res}}$, which means that the microgrid is perfectly able to satisfy the local loads with the locally produced energy, therefore we do not need to add the second condition $N_{\text{gen}}\bar{P}_{\text{dis}} \geq P_{\text{load}} - P_{\text{res}}$, which is instead present in the second column.

Remark 3: Although a rule-based controller design could mean to apply some changes to the rules in case different costs are considered, our proposed approach uses simple rules based on linear inequalities that can easily be verified in real time. We therefore believe that our approach is flexible and can be adapted to cases in which more rules are needed. Moreover, the reduction in the computational complexity achieved, as shown in Section IV, justifies the adoption of this approach. Lastly, an increase in the number of rules will not alter the benefits of adopting this strategy.

Remark 4: Related to the previous remark, it is possible to add rules regarding shut-down in the rule-based approach, if needed. More specifically, since the shut-down costs regard only the generators, one can check if the total shut-down cost is higher than the cost of producing the minimum amount of energy multiplied by the number of time steps during which the generator would be turned off. In other words, if

	$P_{\text{load}} \leq P_{\text{res}}$	$N_{\text{gen}} \bar{P}_{\text{dis}} \geq P_{\text{load}} - P_{\text{res}}$ AND $P_{\text{load}} > P_{\text{res}}$	$N_{\text{gen}} \bar{P}_{\text{dis}} < P_{\text{load}} - P_{\text{res}}$
$c_{\text{pur}} < c_{\text{prod}} < c_{\text{sale}} \leq c_{\text{pur}}$	<u>Case 1:</u> <ul style="list-style-type: none"> • Production units produce energy • Battery can store energy • Grid power exchange: sale enabled 	<u>Case 2:</u> <ul style="list-style-type: none"> • Production units produce energy • Battery can store energy • Grid power exchange: sale enabled 	<u>Case 3:</u> <ul style="list-style-type: none"> • Production units produce energy • Battery provides energy • Grid power exchange: purchase enabled
$c_{\text{pur}} < c_{\text{prod}} \leq c_{\text{prod}} \leq c_{\text{sale}}$	<u>Case 4:</u> <ul style="list-style-type: none"> • Production units are turned off • Battery can store energy • Grid power exchange: sale enabled 	Same as case 2	Same as case 3
$c_{\text{sale}} \leq c_{\text{pur}} \leq c_{\text{prod}} \leq c_{\text{prod}} \leq c_{\text{sale}}$	Same as case 4	<u>Case 5:</u> <ul style="list-style-type: none"> • Production units are turned off • Battery provides energy • Grid power exchange: purchase enabled 	Same as case 5

TABLE I
DIFFERENT CASES OF THE IF-THEN-ELSE RULES FOR THE PROPOSED CONTROLLER

the total shut down cost for generator i from time step k to time step $k + T_{\text{OFF}}$ is $c_{\text{SD}}^i(k)$ and the generator is turned off for at least T_{OFF} time steps, then it is convenient to turn off the generator if $c_{\text{SD}}^i(k) < T_{\text{OFF}} c_{\text{prod}} P_{\text{dis}}$. Since all these quantities are known at time step k , one can check whether the above inequality is true or not.

D. Additional Constraints Required by the Rule-Based Design and Feasibility Issues

When we apply the if-then-rules proposed in the previous Section III-C, we must devote special attention to some of the constraints presented in Section II-C. In particular, the generator constraints (18)-(19) in combination with the power balance constraints (7), (9) pose some issues in some of the cases presented in Table I.

First of all, according to the if-then-else rules proposed in the previous section, it might happen that the generators are imposed to be turned off while according to constraint (19) they should be kept turned on, or vice versa. In this case it is enough to override the proposed if-then-else rules and keep the generators on (vice versa, off) in order to satisfy constraints (18)-(19).

When also taking into account the power balance constraints (7), (9), extra attention is needed. Let us analyze the case in which, at time step \hat{h} , the generators are turned off. Suppose now that at the prediction time step $\hat{h} + 1$ the system is in case 2 of Table I and thus the if-then-else rules procedure would force the generators to be turned on, but constraint (18) is active at $\hat{h} + 1$. According to the rules proposed in Table I, if the generators cannot be turned on due to constraint (18), then the power balance (7) could not be satisfied, since the grid is set to be in the sale mode. In this case, we must then set the grid to the purchase mode.

In all the other cases, the proposed if-then-else rules together with the generator constraints (18)-(19) do not lead to an issue that requires extra attention. In some cases, though, we must verify that some assumptions are guaranteed. For instance, suppose that at the prediction time step $\hat{h} + 1$ the if-then-else rules would make the system switch from case 1 to case 4, but due to constraint (19), the generators cannot be turned off. In this case, the optimization problem is still feasible as long as

$$P_{\text{res}}(\hat{h} + 1) - P_{\text{load}}(\hat{h} + 1) + N_{\text{gen}} P_{\text{dis}} \leq \bar{P}_{\text{grid}}. \quad (22)$$

This constraint is satisfied as long as \bar{P}_{grid} is sufficiently large, which is usually the case in the practical applications. A similar case is verified when the generators are turned off and the if-then-else rules would impose case 3 of Table I, but in this case we would have the less restrictive constraint $P_{\text{res}}(\hat{h} + 1) - P_{\text{load}}(\hat{h} + 1) \leq \bar{P}_{\text{grid}}$.

Lastly, we do not impose the generators to produce at full capacity in cases 2 and 3 since this could lead to infeasibility problems when the generators should be turned off but they remain turned on due to constraint (19).

E. Single-level Two-model Controller

For the overall scheme of our proposed controller we consider a single-level controller rather than a hierarchical one. Our proposed controller uses two different models that have two different sampling times, as explained in Section II-B. At each time step, the controller solves an open-loop optimal control problem, using for predictions the ‘fast’ model until time step $N_{\text{f,s}}$ and the ‘slow’ model until the prediction horizon N_{p} . For both models, the if-then-else rules proposed in Section III-C are applied. After an optimal input sequence

TABLE II
PARAMETERS OF THE MICROGRID USED IN THE CASE STUDY OF SECTION IV.

PARAMETER	VALUE
Maximum ultracapacitor energy level \bar{x}_{uc}	50 [kWh]
Minimum ultracapacitor energy level \underline{x}_{uc}	2 [kWh]
Maximum battery energy level \bar{x}_b	250 [kWh]
Minimum battery energy level \underline{x}_b	25 [kWh]
Battery charging efficiency $\eta_{c,b}$	0.90
Battery discharging efficiency $\eta_{d,b}$	0.90
Ultracapacitor charging efficiency $\eta_{c,uc}$	0.99
Ultracapacitor discharging efficiency $\eta_{d,uc}$	0.99
Maximum interconnection power flow limit \bar{P}_{grid}	1000 [kW]
Minimum interconnection power flow limit \underline{P}_{grid}	-1000 [kW]
Number of generators N_{gen}	3
Maximum power providable by the battery \bar{P}_b	100 [kW]
Maximum power injectable to the battery \underline{P}_b	-100 [kW]
Maximum power providable by the ultracapacitor \bar{P}_{uc}	25 [kW]
Maximum power injectable to the ultracapacitor \underline{P}_{uc}	-25 [kW]
Maximum power level of the dispatchable generators \bar{P}_{dis}	150 [kW]
Minimum power level of the dispatchable generators \underline{P}_{dis}	6 [kW]

is obtained, only the first input of the sequence is applied to the system and at the next time step the procedure is repeated again.

F. Optimization Problem

The overall optimization problem, after we have applied the proposed if-then-else rules of Section III, becomes

$$\min_{\bar{\mathbf{u}}_f} J(\mathbf{P}_p(h), C_{grid}(h)) \quad (23)$$

subject to

dynamics (10), (11), constraints (7), (12) – (19),
parametrization of Section III.

As standard in MPC controllers, we compute the optimal control inputs $\bar{\mathbf{u}}_f$ from the current time step h until time step $h+N_p-1$. We apply only the first element of the optimal input sequence and at the next sampling time we solve problem (23) once again.

IV. SIMULATIONS

In this section, we compare our proposed single-level two-model rule-based MPC controller with a controller that has the same single-level two-model structure but that does not make use of the if-then-else rules proposed in Section III-C and instead uses the standard MILP approach. We denote our proposed rule-based controller as RBMPC, and we denote the other approach by MILP. In both cases we use Gurobi [18] to solve the optimization problems, which are an MILP for the MILP case and a linear programming problem for the RBMPC case.

We simulated different scenarios, with different renewable energy and loads profiles, and compared the two approaches in terms of total computational time and total cost. The profiles are taken from available data at [51]. We used data from the Netherlands, choosing profiles of days between the 1st of May

2018 and the 30th of June 2018. The value of the parameters in the microgrid that we consider are similar to the ones in [11] and are reported in Table II.

Moreover, we also consider different combinations of $T_s, T_f, N_{f,s}, N_p$ and for each of them we performed 20 simulations. Table III summarizes the simulation results and we show, for each different combination, the mean value, maximum value, the minimum value, and the standard deviation of both the total computation time and the overall cost, i.e. the times shown in Table III are total CPU times. Moreover, we also show the average MILP gap for the MILP approach. Note that the maximum value for the MILP gap parameter was set in Gurobi to 10^{-4} . Note also that in the different cases (rows) of Table III we used different days in the dataset that we indicated before, in order to test our method with different renewables and load profiles. In other words, in the different cases of Table III, we did not always choose the data from the same set of days. This also explains why the costs differ considerably in the different rows of Table III. We can see that for the different combinations, the average total cost is very similar for both the approaches and there is at most a 1.2% difference. However, the computational complexity is greatly reduced, since in all the cases the average computational savings are between 65% and 80%. As one could expect, the computational savings are larger when the number of binary variables increases, i.e. when $N_{f,s}$ or N_p , or both, increase. Nevertheless, the performance of our proposed controller does not show a decrease with the number of binary variables.

Figure 3 shows the power flows within the microgrid for a representative scenario, with $N_{f,s} = 12, N_p = 12, T_f = 5\text{min}, T_s = 60\text{min}$, i.e. the scenario in row 4 of Table III. Figure 5 shows, for the same scenario, the evolution of the level of charge of the energy storage systems in the two approaches, while Figure 6 shows the time-varying price profiles. The electricity prices in this scenario are chosen in such a way that the three cases in the rows of Table I are covered. It can be observed that the two different approaches reach a very similar solution. When c_{pur} increases above c_{prod} , all the dispatchable units start producing power to satisfy the loads. Moreover, during the peak production hours of renewable energy sources, when $c_{prod} < c_{pur}$, both methods still produce energy and they sell the excess energy to the main grid. What is different in the two approaches is that the MILP algorithm charges the battery more often compared to the RBMPC controller and it also utilizes more the ESSs. Apart from this, the two solutions are very similar. Indeed, Table IV shows the comparison of different quantities for the selected scenario, comparing the total produced power with the dispatchable generators P_{dis}^{TOT} , the power acquired from the main grid $P_{grid,purchased}^{TOT}$, and the power sold to the main grid $P_{grid,sold}^{TOT}$, and it can be noted that there is almost no difference between the two approaches.

Note that in some cases, i.e. rows 4 and 5, the RBMPC approach achieves a lower cost than the MILP one. This could in theory not be possible, since the RBMPC approach only considers one case (due to the fact that binary variables are assigned) while the MILP one considers many more cases for the values of the binary variables. This implies that the cost of the MILP approach should be always lower than or equal to

$N_{f,s}, T_f$	T_s	N_p	Total CPU Time MILP [s]	Total CPU Time RBMPC [s]	Cost MILP [€]	Cost RBMPC [€]
$N_{f,s} = 6$ $T_f = 5\text{min}$	30min	6	$\mu = 35.85$ $\sigma = 4.39$ $\Delta = 44.20$ $\nabla = 27.78$ GAP = $0.438 \cdot 10^{-5}$	$\mu = 12.58$ $\sigma = 0.94$ $\Delta = 14.56$ $\nabla = 11.40$ (-65%)	$\mu = 5491$ $\sigma = 1174$ $\Delta = 8680$ $\nabla = 3308$	$\mu = 5494$ $\sigma = 1184$ $\Delta = 8703$ $\nabla = 3297$ (+0.1%)
		24	$\mu = 40.69$ $\sigma = 9.10$ $\Delta = 52.87$ $\nabla = 18.83$ GAP = $1.818 \cdot 10^{-5}$	$\mu = 12.99$ $\sigma = 2.25$ $\Delta = 15.80$ $\nabla = 10.09$ (-68%)	$\mu = 5896$ $\sigma = 1532$ $\Delta = 9072$ $\nabla = 3292$	$\mu = 5899$ $\sigma = 1540$ $\Delta = 9087$ $\nabla = 3286$ (+0.0%)
		48	$\mu = 54.59$ $\sigma = 12.77$ $\Delta = 81.33$ $\nabla = 19.04$ GAP = $2.295 \cdot 10^{-5}$	$\mu = 12.79$ $\sigma = 2.82$ $\Delta = 20.38$ $\nabla = 10.88$ (-77%)	$\mu = 5533$ $\sigma = 1453$ $\Delta = 8811$ $\nabla = 3311$	$\mu = 5589$ $\sigma = 1455$ $\Delta = 8870$ $\nabla = 3356$ (+1.0%)
$N_{f,s} = 12$ $T_f = 5\text{min}$	60min	12	$\mu = 38.19$ $\sigma = 3.12$ $\Delta = 47.37$ $\nabla = 33.34$ GAP = $1.720 \cdot 10^{-5}$	$\mu = 11.07$ $\sigma = 0.47$ $\Delta = 11.82$ $\nabla = 10.39$ (-71%)	$\mu = 5808$ $\sigma = 1006$ $\Delta = 7516$ $\nabla = 3735$	$\mu = 5801$ $\sigma = 1010$ $\Delta = 7497$ $\nabla = 3722$ (-0.1%)
$N_{f,s} = 30$ $T_f = 1\text{min}$	30min	6	$\mu = 304.12$ $\sigma = 48.11$ $\Delta = 364.93$ $\nabla = 203.90$ GAP = $0.563 \cdot 10^{-5}$	$\mu = 103.97$ $\sigma = 13.95$ $\Delta = 112.62$ $\nabla = 68.97$ (-66%)	$\mu = 5837$ $\sigma = 1188$ $\Delta = 9207$ $\nabla = 3880$	$\mu = 5825$ $\sigma = 1183$ $\Delta = 9150$ $\nabla = 3846$ (-0.2%)
		24	$\mu = 383.92$ $\sigma = 59.34$ $\Delta = 464.99$ $\nabla = 238.43$ GAP = $1.927 \cdot 10^{-5}$	$\mu = 119.71$ $\sigma = 17.03$ $\Delta = 131.04$ $\nabla = 80.87$ (-69%)	$\mu = 6252$ $\sigma = 1097$ $\Delta = 8375$ $\nabla = 3885$	$\mu = 6262$ $\sigma = 1105$ $\Delta = 8373$ $\nabla = 3846$ (+0.2%)
		48	$\mu = 368.46$ $\sigma = 84.26$ $\Delta = 627.44$ $\nabla = 182.03$ GAP = $2.504 \cdot 10^{-5}$	$\mu = 72.59$ $\sigma = 6.49$ $\Delta = 79.45$ $\nabla = 61.53$ (-80%)	$\mu = 5635$ $\sigma = 1543$ $\Delta = 9067$ $\nabla = 3310$	$\mu = 5703$ $\sigma = 1550$ $\Delta = 9109$ $\nabla = 3337$ (+1.2%)
$N_{f,s} = 60$ $T_f = 1\text{min}$	60min	3	$\mu = 393.95$ $\sigma = 82.72$ $\Delta = 479.61$ $\nabla = 190.30$ GAP = $0.828 \cdot 10^{-5}$	$\mu = 107.63$ $\sigma = 3.98$ $\Delta = 111.56$ $\nabla = 99.14$ (-73%)	$\mu = 5671$ $\sigma = 1534$ $\Delta = 9063$ $\nabla = 3295$	$\mu = 5676$ $\sigma = 1541$ $\Delta = 9087$ $\nabla = 3292$ (+0.1%)
		12	$\mu = 452.62$ $\sigma = 98.51$ $\Delta = 605.06$ $\nabla = 201.17$ GAP = $1.832 \cdot 10^{-5}$	$\mu = 102.37$ $\sigma = 12.75$ $\Delta = 112.44$ $\nabla = 66.27$ (-77%)	$\mu = 5683$ $\sigma = 1335$ $\Delta = 8932$ $\nabla = 3375$	$\mu = 5695$ $\sigma = 1343$ $\Delta = 8893$ $\nabla = 3332$ (+0.2%)

TABLE III

SIMULATION RESULTS FOR DIFFERENT SAMPLING TIMES AND PREDICTION HORIZONS. THE SYMBOLS μ , σ , Δ , AND ∇ INDICATE THE MEAN, STANDARD DEVIATION, MAXIMUM, AND MINIMUM RESPECTIVELY, WHILE 'GAP' INDICATES THE AVERAGE MILP GAP. IN COLUMNS 5 AND 7 THE DIFFERENCE OF PERFORMANCE IN PERCENTAGE BETWEEN THE MILP AND RBMPC APPROACH IS SHOWN.

the one of the RBMPC approach. However, it can happen that the MILP approach decides to charge the battery more and, at the end of the simulation, the stored energy in the battery could be larger than the energy stored in the battery with the RBMPC approach. This means that there is extra energy that has not been used or sold. Since the cost used in Table III does not consider a terminal cost, it can then happen that the RBMPC approach has a lower cost than the MILP approach, but this implies that a higher amount of energy is stored in the battery in the MILP approach at the end of the simulation.

Note also that in some cases of Table III the minimum value for the CPU time of the MILP case is smaller than the maximum value of the CPU time of the rule-based case. This does not mean that the MILP was able to find a solution before the rule-based approach in that specific simulation. Indeed, this is due to the fact that many simulations were performed for each row and for certain profiles of P_{res} and P_{load} the solver could find a solution in less time, both for the MILP and

RBMPC case. However, in each single simulation the rule-based approach had a smaller computation time compared to the MILP one.

V. CONCLUSIONS

We have presented a rule-based MPC controller for the optimization of the operation of a grid-connected microgrid. The controller follows if-then-else rules in order to assign values to the binary variables in the hybrid model of the microgrid, e.g. ON or OFF status of the generators, charging or discharging modes of the batteries. Moreover, a single-level controller structure is used, that relies on adopting two different models of the microgrid that have different number of states and sampling times for predictions in the MPC optimization procedure. The proposed approach, compared to the standard one available in the literature, requires a considerably lower amount of computation time. Indeed, while the standard approach from literature is based on an MILP

TABLE IV

SIMULATION RESULTS COMPARISON BETWEEN RBMPC AND MILP FOR A SCENARIO WITH $N_{f,s} = 12$, $N_p = 12$, $T_f = 5\text{min}$, $T_s = 60\text{min}$. THE TOTAL POWER PRODUCED BY DISPATCHABLE GENERATORS IS REPRESENTED BY $P_{\text{dis}}^{\text{TOT}}$, WHILE $P_{\text{g,sold}}^{\text{TOT}}$ IS THE TOTAL POWER SOLD TO THE GRID AND $P_{\text{g,purchased}}^{\text{TOT}}$ IS THE TOTAL POWER BOUGHT FROM THE MAIN GRID.

	$P_{\text{p}}^{\text{TOT}}$ [kW]	$P_{\text{grid,sold}}^{\text{TOT}}$ [kW]	$P_{\text{grid,purchased}}^{\text{TOT}}$ [kW]	Total cost [€]
MILP	88557	31687	52012	4059.40
RBMPC	88726 (+0.19%)	31335 (-1.11%)	51855(-0.30%)	4081.00 (+0.53%)

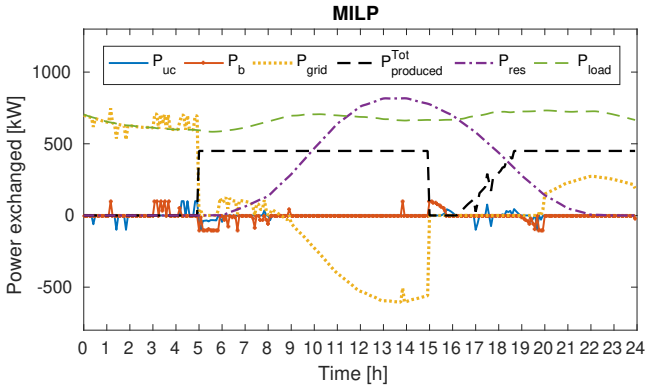
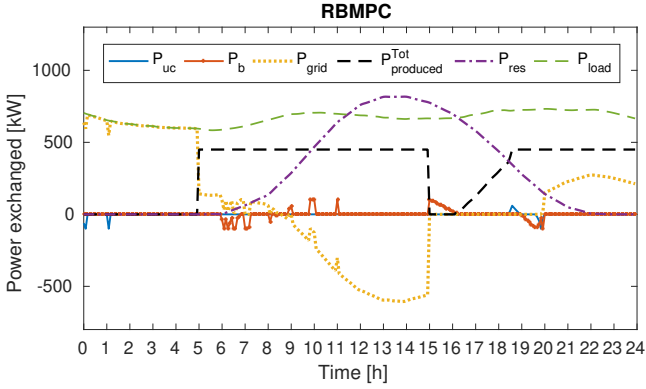


Fig. 3. Power flows in the microgrid during the considered simulation, when the RBMPC controller is used (top) and when an MILP controller is applied (bottom) with $N_{f,s} = 12$, $N_p = 12$, $T_f = 5\text{min}$, $T_s = 60\text{min}$.

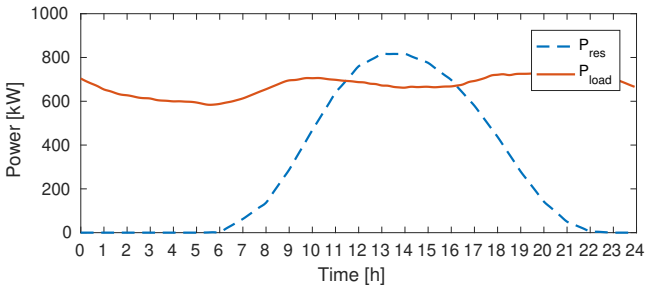


Fig. 4. Load profile (P_{load}) and renewable sources generation profile (P_{res}) in the considered simulation, with $N_{f,s} = 12$, $N_p = 12$, $T_f = 5\text{min}$, $T_s = 60\text{min}$.

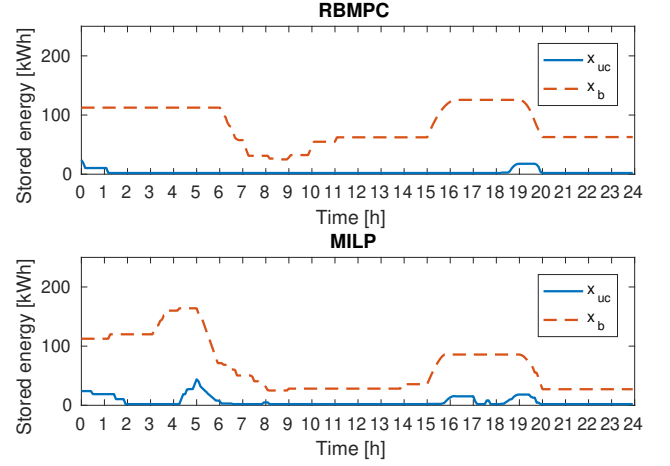


Fig. 5. Stored energy in the storage devices when the RBMPC controller is used (top) and when an MILP controller is applied (bottom), with $N_{f,s} = 12$, $N_p = 12$, $T_f = 5\text{min}$, $T_s = 60\text{min}$.

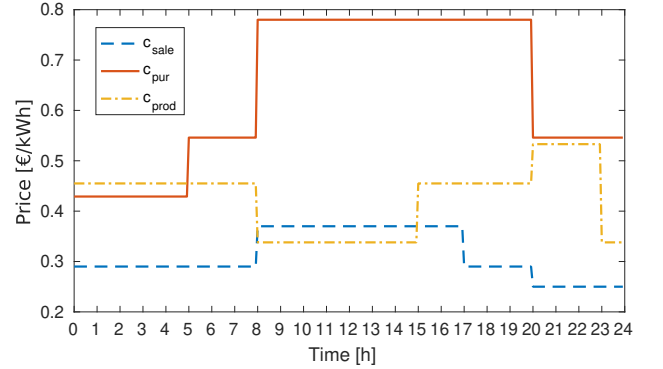


Fig. 6. Electricity purchase (c_{pur}), sale (c_{sale}), and production (c_{prod}) prices in the considered simulations of Table III.

problem, we are able to convert it to a linear programming problem. The simulations carried out show that with our proposed method we can reduce the computational complexity up to around one fourth of the time required by the other approach, with almost no loss on performance. However, the amount of computational savings is expected to increase when the number of binary variables is larger. Therefore, our method provides a way to drastically reduce the computational complexity when the number of binary variables involved is large; the proposed method can be thus applied in a real microgrid when the amount of computation time and/or power is limited.

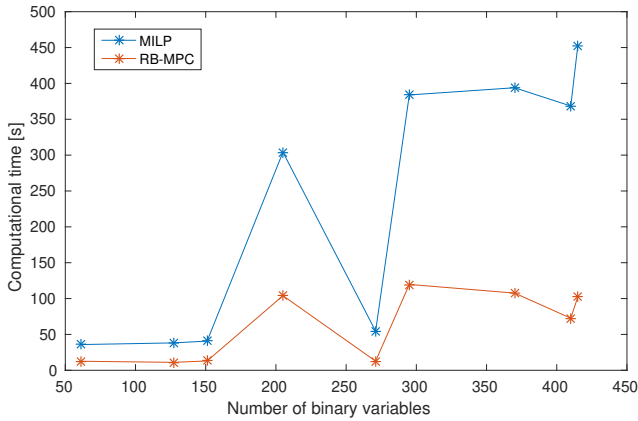


Fig. 7. Average computation times of the MILP and RB-MPC approaches as a function of the number of variables.

As a first step for extending the current work, we will consider if-then-else rules that make use of the future values of the electricity prices and not of the current one only. We will also consider a set of rules to include flexible loads management and demand side management policies. Furthermore, we will analyze which is the impact of some specific inter-temporal constraints on the overall computation time, i.e. constraints (18), (19) related to the minimum amount of time during which the generators should stay turned on or off. Moreover, the proposed if-then-else rules are obtained heuristically in this article, but as future work we will study a systematic approach to obtain the rules for assigning the values to the binary decision variables. We also suggest, as future work, a comparison between the standard MILP method and the proposed one on real test scenarios, e.g. using a microgrid lab setup as the one considered in [15], possibly together with a decentralized optimization approach using decomposed or distributed optimization methods. Lastly, we will consider an extension of our work for the case of a microgrid in islanded mode.

REFERENCES

- [1] European Parliament. Directive 2009/28/EC on the promotion of the use of energy from renewable sources. Official Journal of the European Union, 2009.
- [2] X. Fang, S. Misra, G. Xue, and D. Yang. Smart grid — the new and improved power grid: A survey. *IEEE Communications Surveys Tutorials*, 14(4):944–980, 2012.
- [3] S. Parhizi, H. Lotfi, A. Khodaei, and S. Bahramirad. State of the art in research on microgrids: A review. *IEEE Access*, 3:890–925, 2015.
- [4] H. Jiayi, J. Chuanwen, and X. Rong. A review on distributed energy resources and microgrid. *Renewable and Sustainable Energy Reviews*, 12(9):2472–2483, 2008.
- [5] N. Hatziaargyriou, H. Asano, R. Iravani, and C. Marnay. Microgrids. *IEEE Power and Energy Magazine*, 5(4):78–94, 2007.
- [6] B. Zhao, X. Wang, D. Lin, M. M. Calvin, J. C. Morgan, R. Qin, and C. Wang. Energy management of multiple microgrids based on a system of systems architecture. *IEEE Transactions on Power Systems*, 33(6):6410–6421, 2018.
- [7] T. Zhao and Z. Ding. Distributed agent consensus-based optimal resource management for microgrids. *IEEE Transactions on Sustainable Energy*, 9(1):443–452, 2018.
- [8] W. Shi, N. Li, C. Chu, and R. Gadh. Real-time energy management in microgrids. *IEEE Transactions on Smart Grid*, 8(1):228–238, 2017.
- [9] Y. Zheng, Y. Song, D. J. Hill, and Y. Zhang. Multiagent system based microgrid energy management via asynchronous consensus ADMM. *IEEE Transactions on Energy Conversion*, 33(2):886–888, 2018.
- [10] C. Ju, P. Wang, L. Goel, and Y. Xu. A two-layer energy management system for microgrids with hybrid energy storage considering degradation costs. *IEEE Transactions on Smart Grid*, 9(6):6047–6057, 2018.
- [11] A. Parisio, E. Rikos, and L. Glielmo. A model predictive control approach to microgrid operation optimization. *IEEE Transactions on Control Systems Technology*, 22(5):1813–1827, 2014.
- [12] A. Parisio, E. Rikos, and L. Glielmo. Stochastic model predictive control for economic/environmental operation management of microgrids: An experimental case study. *Journal of Process Control*, 43:24–37, 2016.
- [13] S. Raimondi Cominesi, M. Farina, L. Giulioni, B. Picasso, and R. Scatoloni. A two-layer stochastic model predictive control scheme for microgrids. *IEEE Transactions on Control Systems Technology*, 26(1):1–13, 2018.
- [14] T. Pippia, J. Sijs, and B. De Schutter. A parametrized model predictive control approach for microgrids. In *2018 IEEE Conference on Decision and Control (CDC)*, pages 3171–3176, 2018.
- [15] P. Velarde, L. Valverde, J. M. Maestre, C. Ocampo-Martinez, and C. Bordons. On the comparison of stochastic model predictive control strategies applied to a hydrogen-based microgrid. *Journal of Power Sources*, 343:161–173, 2017.
- [16] F. Berkel, D. Görge, and S. Liu. Load-frequency control, economic dispatch and unit commitment in smart microgrids based on hierarchical model predictive control. In *52nd IEEE Conference on Decision and Control*, pages 2326–2333, 2013.
- [17] J. Sachs and O. Sawodny. A two-stage model predictive control strategy for economic diesel-PV-battery island microgrid operation in rural areas. *IEEE Transactions on Sustainable Energy*, 7(3):903–913, 2016.
- [18] Gurobi Optimization, Inc. Gurobi optimizer reference manual. <http://www.gurobi.com>, 2016.
- [19] IBM ILOG CPLEX Optimizer. <http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/>, 2010.
- [20] C. A. Floudas. *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications*. Topics in Chemical Engineering. Oxford University Press, 1995.
- [21] M. Conforti, G. Cornuejols, and G. Zambelli. *Integer Programming*. Springer Publishing Company, Incorporated, 2014.
- [22] A. Schrijver. *Theory of Linear and Integer Programming*. John Wiley & Sons, Chichester, UK, 1986.
- [23] D. R. Morrison, S. H. Jacobson, J. J. Sauppe, and E. C. Sewell. Branch-and-bound algorithms: A survey of recent advances in searching, branching, and pruning. *Discrete Optimization*, 19:79–102, 2016.
- [24] J. F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4(1):238–252, 1962.
- [25] A. M. Geoffrion. Generalized Benders decomposition. *Journal of Optimization Theory and Applications*, 10(4):237–260, 1972.
- [26] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsbergh, and P. H. Vance. Branch-and-price: Column generation for solving huge integer programs. *Operations Research*, 46(3):316–329, 1998.
- [27] M. E. Lübbecke and J. Desrosiers. Selected topics in column generation. *Operations Research*, 53(6):1007–1023, 2005.
- [28] G. B. Dantzig and P. Wolfe. Decomposition principle for linear programs. *Operations Research*, 8(1):101–111, 1960.
- [29] S. K. Zegeye, B. De Schutter, J. Hellendoorn, E. A. Breunnesse, and A. Hegyi. A predictive traffic controller for sustainable mobility using parameterized control policies. *IEEE Transactions on Intelligent Transportation Systems*, 13(3):1420–1429, 2012.
- [30] F. Oldewurtel, A. Parisio, C. N. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari. Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings*, 45:15–27, 2012.
- [31] C. Wang, C. J. Ong, and M. Sim. Model predictive control using segregated disturbance feedback. *IEEE Transactions on Automatic Control*, 55(4):831–840, 2010.
- [32] C. H. Papadimitriou. On the complexity of integer programming. *Journal of the ACM*, 28(4):765–768, 1981.
- [33] F. Alavi, N. van de Wouw, and B. De Schutter. Min-max control of fuel-cell-car-based smart energy systems. In *2016 European Control Conference (ECC)*, pages 1223–1228, 2016.
- [34] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.
- [35] J. P. Fossati, A. Galarza, A. Martín-Villate, and L. Fontán. A method for optimal sizing energy storage systems for microgrids. *Renewable Energy*, 77:539–549, 2015.

- [36] E. Ghiani, C. Vertuccio, and F. Pilo. Optimal sizing and management of a smart microgrid for prevailing self-consumption. In *2015 IEEE Eindhoven PowerTech*, pages 1–6, 2015.
- [37] N. G. Paterakis, I. N. Pappi, O. Erdinç, R. Godina, E. M. G. Rodrigues, and J. P. S. Catalão. Consideration of the impacts of a smart neighborhood load on transformer aging. *IEEE Transactions on Smart Grid*, 7(6):2793–2802, 2016.
- [38] L. Igualada, C. Corchero, M. Cruz-Zambrano, and F. J. Heredia. Optimal energy management for a residential microgrid including a vehicle-to-grid system. *IEEE Transactions on Smart Grid*, 5(4):2163–2172, 2014.
- [39] S. Aslam, N. Javaid, F. A. Khan, A. Alamri, A. Almogren, and W. Abdul. Towards efficient energy management and power trading in a residential area via integrating a grid-connected microgrid. *Sustainability*, 10(4), 2018.
- [40] P. Malysz, S. Sirouspour, and A. Emadi. An optimal energy storage control strategy for grid-connected microgrids. *IEEE Transactions on Smart Grid*, 5(4):1785–1796, 2014.
- [41] D. Arcos-Aviles, J. Pascual, F. Guinjoan, L. Marroyo, P. Sanchis, and M. P. Marietta. Low complexity energy management strategy for grid profile smoothing of a residential grid-connected microgrid using generation and demand forecasting. *Applied Energy*, 205:69–84, 2017.
- [42] R. Majumder, A. Ghosh, G. Ledwich, and F. Zare. Power management and power flow control with back-to-back converters in a utility connected microgrid. *IEEE Transactions on Power Systems*, 25(2):821–834, 2010.
- [43] L. Ma, N. Liu, J. Zhang, W. Tushar, and C. Yuen. Energy management for joint operation of CHP and PV prosumers inside a grid-connected microgrid: A game theoretic approach. *IEEE Transactions on Industrial Informatics*, 12(5):1930–1942, 2016.
- [44] W. Feng, K. Sun, Y. Guan, J. M. Guerrero, and X. Xiao. Active power quality improvement strategy for grid-connected microgrid based on hierarchical control. *IEEE Transactions on Smart Grid*, 9(4):3486–3495, 2018.
- [45] M. A. Abusara, S. M. Sharkh, and J. M. Guerrero. Improved droop control strategy for grid-connected inverters. *Sustainable Energy, Grids and Networks*, 1:10–19, 2015.
- [46] B. Hredzak, V. G. Agelidis, and M. Jang. A model predictive control system for a hybrid battery-ultracapacitor power source. *IEEE Transactions on Power Electronics*, 29(3):1469–1479, 2014.
- [47] E. F. Camacho and C. B. Alba. *Model Predictive Control*. Advanced Textbooks in Control and Signal Processing. Springer London, 2013.
- [48] D. Q. Mayne. Model predictive control: Recent developments and future promise. *Automatica*, 50(12):2967–2986, 2014.
- [49] J.M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, England., 2002.
- [50] E. F. Camacho, D. R. Ramirez, D. Limon, D. Muñoz de la Peña, and T. Alamo. Model predictive control techniques for hybrid systems. *Annual Reviews in Control*, 34(1):21–31, 2010.
- [51] ENTSO-E, the European Network of Transmission System Operators. <https://transparency.entsoe.eu/>.