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J. R. D. Frejo, I. Papamichail, M. Papageorgiou, and B. De Schutter

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Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: <https://www.dcsc.tudelft.nl>

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A new macroscopic model for Variable Speed Limits

Jose Ramon D. Frejo* Ioannis Papamichail**
Markos Papageorgiou** Bart De Schutter*

* *Delft Center for Systems and Control, Delft University of Technology, The Netherlands (e-mails: J.R.DominguezFrejo@tudelft.nl, B.DeSchutter@tudelft.nl)*

** *Dynamic Systems and Simulation Laboratory, Technical University of Crete, Greece. (e-mails: ipapa@dssl.tuc.gr, markos@dssl.tuc.gr)*

Abstract: This paper proposes a new macroscopic model for Variable Speed Limits (VSLs), combining characteristics of previously proposed models, in order to have the capability of modeling different capacities, critical densities, and levels of compliance for links affected by speed limits. Moreover, the effects of VSLs on the fundamental diagram of traffic flow are studied concluding that, at least for the considered stretch of the A12 freeway in The Netherlands, the capacity of a freeway link is decreased (and the critical density is increased) by reducing the value of the corresponding speed limit. Finally, it is shown that the VSL-induced fundamental diagram is not triangular and that the speed limit compliance can be very low if enforcement measures are not applied.

Keywords: Variable Speed Limit, Freeway Traffic Control, Freeway Traffic Modeling

1. INTRODUCTION

In the past years, Variable Speed Limits (VSLs) have emerged as a potential traffic management measure for increasing freeway efficiency. Nowadays, two kinds of VSL control algorithms are considered in the literature:

- **Safety-oriented VSLs:** This kind of VSL control algorithms are applied in order to increase traffic safety by reducing the speed limit when a vehicle is approaching a congested link or an incident. However, these applications in general do not result in a significant improvement of flow or capacity.
- **VSLs for traffic efficiency improvement:** The goal of these VSL control algorithms is to avoid, resolve, or reduce traffic jams on freeways by limiting the arriving flow and, thereby, avoiding or mitigating the capacity drop (Hegyi and Hoogendoorn (2010); Carlson et al. (2010, 2011)). These techniques have been mainly tested by simulation and, therefore, their potential improvement in the traffic conditions relies on the model used. Moreover, the use of model-based optimal control algorithms like Model Predictive Control can considerably improve the Total Time Spent (TTS) and other traffic performance indices, but their success highly depends on model accuracy (Hegyi et al. (2005); Frejo and Camacho (2012)).

The main goal of this paper is to propose a new model for VSLs, combining characteristics of the previously proposed models, in order to have a better fit of the Fundamental Diagram traffic data. Moreover, field data available from the SPECIALIST field test (Hegyi and Hoogendoorn (2010)) are used to analyze the effects of VSLs on the capacity and the critical density of a link, and to study the effects of VSLs on the speeds of uncongested links.

The paper is structured as follows. Section 2 introduces the two main ways of modeling VSLs within the macroscopic model METANET. Section 3 shows the newly proposed model for VSLs. Section 4 summarizes the main characteristics of the data set used. Sections 5 and 6 analyze, respectively, the capacity reduction and the free flow response induced by the VSLs based on Fundamental Diagram (FD) measurements. Finally, conclusions are drawn in Section 7.

2. MACROSCOPIC VSL MODELING

2.1 Traffic models

Two main approaches are being used for traffic flow modeling: microscopic models (which describe the longitudinal and lateral movement of individual vehicles) and macroscopic models (which mostly model traffic as a particular fluid with aggregate variables such as density, mean speed and flow). This paper focuses on the modeling of VSLs within the framework of macroscopic traffic models, because these models are more suitable for real-time applications than microscopic models. Within macroscopic traffic flow models, first-order models like the Cell Transmission Model (CTM) (Daganzo (1994)) include a static speed-density relationship. On the other hand, second-order models like METANET (Papageorgiou et al. (2010))

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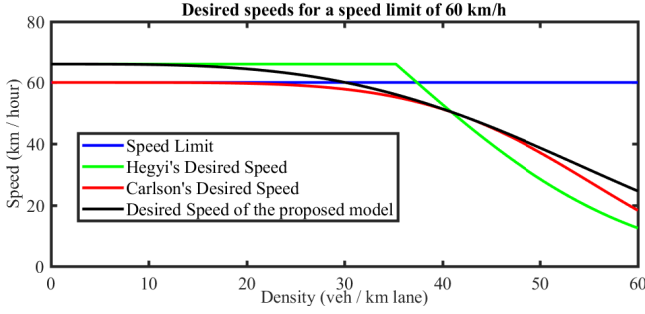


Fig. 1. Desired speeds for $v_{f,i} = 120$ km/h, $\rho_{c,i} = 30$ veh/(km lane), $a_i = 2.5$, $V_{c,i}(k) = 60$ km/h, $\alpha_i = 0.1$, $A_i = 0.8$, $E_i = 3$ (for Carlson's model) and $E_i = 1.66$ (for the proposed model).

address the speed as another state variable with an according state equation, which is capable of capturing additional dynamics and, more important, is able to model capacity drop.

We have focused this work on the modeling of VSL by modifying the second-order model METANET. In any case, the analyzed models for the FD induced by the VSLs could be included within other 1st and 2nd order macroscopic model. For instance, the CTM model could be modified by adjusting the supply and demand functions.

2.2 METANET

Two main equations describe the system dynamics of METANET model. The first one expresses the conservation of vehicles while the second equation expresses the mean speed as a sum of the previous mean speed, a relaxation term, a convection term, and an anticipation term (Papageorgiou et al. (2010)). The traffic behavior is mainly influenced by the fundamental diagram that is expressed by the following equation for a freeway link i :

$$V_i(k) = v_{f,i} \exp\left(-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{c,i}}\right)^{a_i}\right) \quad (1)$$

where $V_i(k)$ is the desired speed for the drivers in absence of a VSL, a_i is a model parameter, $v_{f,i}$ is the free flow speed that the vehicles reach at zero density, $\rho_i(k)$ is the density of the link, and $\rho_{c,i}$ is the critical density (i.e. the density corresponding to the maximum flow in the fundamental diagram).

2.3 VSL model of Hegyi et al.

In the literature, two main ways of including the effects of VSLs in METANET have been considered. The first one was proposed by Hegyi et al. (2004). In this model, VSLs are included in the model by modifying the desired speed equation. When a VSL is applied, the desired speed is computed by taking the minimum of two quantities: the speed based on the prevailing traffic conditions, and the speed caused by the VSL (affected by a compliance term):

$$V_i(k) = \min\left(v_{f,i} \exp\left(-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{c,i}}\right)^{a_i}\right), (1 + \alpha_i) V_{c,i}(k)\right) \quad (2)$$

where $V_{c,i}(k)$ is the value of the VSL on link i , and α_i is a model parameter that reflects the driver compliance.

2.4 VSL model of Carlson et al.

The second model that is commonly used in the literature was proposed by Papamichail et al. (2008) and it has been mainly used by Carlson et al. (2010, 2011). In this model, FD parameters are modified to incorporate the influence of the VSLs. The VSL rate $b_i(k)$ (defined between 0 and 1) is used instead of the implemented value of the VSL $V_{c,i}(k)$:

$$\begin{aligned} V_i(k) &= v_{f,i}^*(k) \exp\left(-\frac{1}{a_i^*(k)} \left(\frac{\rho_i(k)}{\rho_{c,i}^*(k)}\right)^{a_i^*(k)}\right) \\ b_i(k) &= \frac{V_{c,i}(k)}{V_{c,i}^{\max}(k)} \\ v_{f,i}^*(k) &= v_{f,i} b_i(k) \\ \rho_{c,i}^*(k) &= \rho_{c,i} (1 + A_i (1 - b_i(k))) \\ a_i^*(k) &= a_i (E_i - (E_i - 1) b_i(k)) \end{aligned} \quad (3)$$

3. NEW PROPOSED MODEL

3.1 Model formulation

In this paper, a new model is proposed combining the advantages of both approaches. The effect of a VSL is modeled by modifying the FD parameters including a calibration parameter for each equation (α_i for $v_{f,i}$, A_i for $\rho_{c,i}$, and E_i for a_i):

$$\begin{aligned} V_i(k) &= v_{f,i}^*(k) \exp\left(-\frac{1}{a_i^*(k)} \left(\frac{\rho_i(k)}{\rho_{c,i}^*(k)}\right)^{a_i^*(k)}\right) \\ b_i^r(k) &= \min\left(\frac{V_{c,i}(k)}{V_{c,i}^{\max}(k)} (1 + \alpha_i), 1\right) \\ v_{f,i}^*(k) &= \min(V_{c,i}^{\max}(k) b_i^r(k), v_{f,i}) \\ \rho_{c,i}^*(k) &= \rho_{c,i} (1 + A_i (1 - b_i^r(k))) \\ a_i^*(k) &= a_i (E_i - (E_i - 1) b_i^r(k)) \end{aligned} \quad (4)$$

Moreover, in contrast to Carlson's model, the VSL-induced free-flow speed depends on the maximum value of the VSL and does not depend on the free-flow speed without VSL. This definition is more realistic for links that have a maximum value for the VSL that is larger than the maximum free-flow speed. For example, for a link with full compliance, a maximum speed limit of 120 km/h and a free-flow speed of 100 km/h, the VSL-induced free-flow speed for a speed limit of 60 km/h should be 60 km/h (and not 50 km/h).

3.2 Model comparison based on Fundamental Diagrams and desired speeds

An example of the desired speed and FD obtained with the proposed model and with Hegyi's and Carlson's models can be seen in Fig. 1 and 2.

According to the formulation of the models, the three characteristics parameters of the FD change as follows:

- **VSL-induced free-flow speed:** Using Hegyi's model and the proposed model, compliance can be adjusted by the parameter α ; hence, the VSL-induced free flow speed can be freely set according to real data as shown in (2) and (4).

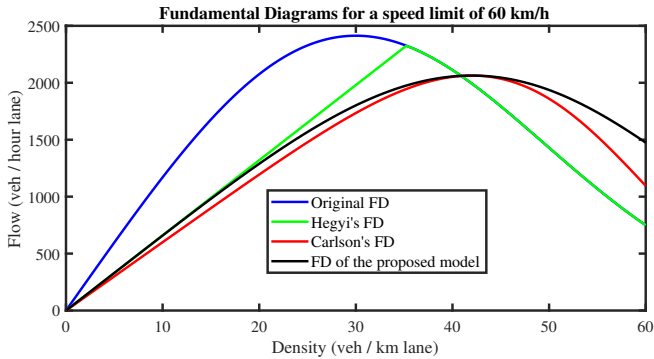


Fig. 2. FD with the same parameters as Fig. 1.

On the other hand, compliance is not included in Carlson's model because it is assumed that the VSL-induced free-flow speed is equal to the value of the corresponding VSL (i.e. for steady states with low densities, the mean speed of a link will be equal the current VSL value in the corresponding link).

It is noted that, using the proposed model, compliance is also affecting the VSL-induced critical density and capacity. This allows to adapt an already calibrated model if the compliance changes (e.g. if the speed limit compliance is enforced by fines or other measures).

In Fig. 3, the FDs and the desired speeds obtained for different values of the compliance α are shown. The figure is equivalent to the one that would be obtained for the implementation of different values of the speed limit (with the same compliance). It can be seen that, as expected from real traffic, the influence of the speed limits (and their compliance) is very low when the system is considerably congested.

- **VSL-induced critical density:** Using Carlson's model and the proposed model, the critical density induced by a VSL can be adjusted in order to approximate real data by changing the parameter A_i as shown in (3) and (4).

On the other hand, using Hegyi's model, the VSL-induced critical density cannot be adjusted for a given value of compliance and the original FD.

- **VSL-induced capacity:** Using Carlson's model and the proposed model, the VSL-induced capacity can be also adjusted in order to fit measurements by using

the parameters E_i and A_i . Nevertheless, using Hegyi's model the VSL-induced capacity is fixed for a given value of α .

In summary, Hegyi's model can be adapted for different VSL-induced free flow speeds, but it lacks the capability of modeling different VSL-induced capacity reductions and critical densities while Carlson's model is able to model different VSL-induced capacities and critical densities, but it does not consider different levels of compliance. On the other hand, the newly proposed model can be adapted for different VSL-induced free-flow speeds, capacities, and critical densities.

4. DATA SET

The data used in this paper comes from the application of the SPECIALIST algorithm on a part of the Dutch A12 freeway (Hegyi and Hoogendoorn (2010)), between the connection with the N11 at Bodegraven up to Harmelen.

The freeway is equipped with variable message signs, that display the current VSL, located at the beginning of each link. Under each gantry there are also double-loop detectors (one pair for each lane), measuring speeds and flows. The data set used includes speed and flow measurements with a sampling time of 10 s (computed as a moving average of the last minute) collected during six months (from September 2009 to February 2010). The data set also includes the speed limit values that have been shown on the variable message signs. The possible values of displayed variable speed limits are 50, 60, 70, 80, 90, 100, and 120 km/h.

The aggregated flow of each link is directly taken from the corresponding loop detector at the beginning of the link while the aggregated speed is the arithmetic mean of the speeds measured in the loop detectors at the beginning and at the end of each link.

The network does not show recurrent congestion from a bottleneck. The most common traffic jams appearing on the freeway are shock waves originating from further downstream of the freeway stretch. For some days, the freeway stretch also shows non-recurrent congestion caused by bottlenecks.

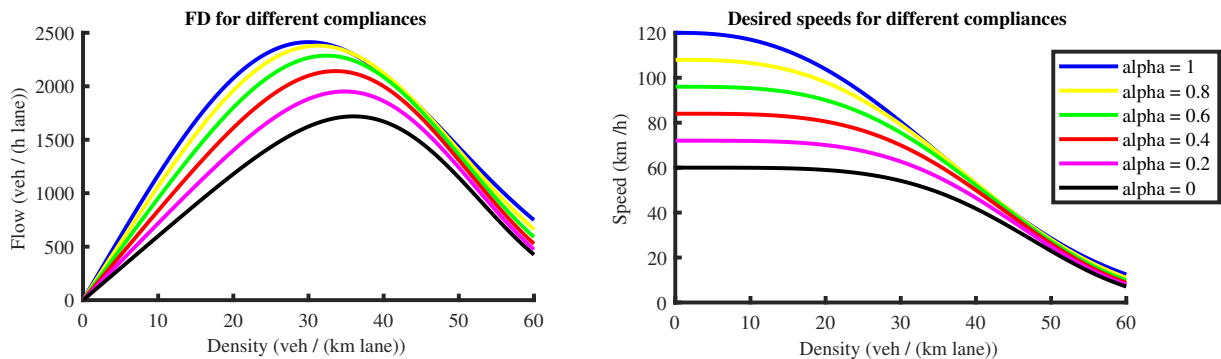


Fig. 3. FD and desired speed for different levels of compliance using the proposed model with $v_{f,i} = 120$ km/h, $\rho_{c,i} = 30$ veh/(km lane), $a_i = 2.5$, $V_{c,i}(k) = 60$ km/h, $\alpha_i = 0.1$, $A_i = 0.4$, and $E_i = 2.5$.

5. VSL-INDUCED CAPACITY AND CRITICAL DENSITY

Previous references have found that the critical density is increased when speed limits are decreased (Papageorgiou et al. (2008); Soriguera et al. (2016, 2017); Van den Hoogen and Smulders (1994)). On the other hand, different conclusions have been drawn about the potential reduction (or increase) of the capacity of a link affected by a VSL (Van den Hoogen and Smulders (1994); Papageorgiou et al. (2008); Soriguera et al. (2016, 2017)).

When analyzing the capacity of a link under different speed limits, the following considerations should be taken into account:

- The analyzed link must be an active bottleneck of a traffic jam.
- It is necessary to have data with two different speed limits at the time that the considered bottleneck reaches capacity.
- It is preferable to use data with constant speed limits (while reaching congestion) in order to avoid dynamic effects that may hinder a proper capacity estimation.

With the exception of the study in Soriguera et al. (2016), the research works previously published do not completely fulfill these requirements for a proper capacity estimation. Within the A12 dataset considered in this study, one link of the freeway fulfills the three mentioned conditions: A bottleneck (on kilometer point 44,7) that has different (but constant during long periods of time) values for the VSLs while reaching congestion. In order to perform a fair analysis, the days for which only one speed limit is used and the days with a much lower free flow speed during long periods of the day (due to weather conditions) are not used. As a result, 34 days have been considered for this study. Furthermore, in order to only consider congestion created by the own link (for which capacity can be estimated), the traffic jams coming from downstream links are also left out of the study of this section. The

resulting measurements for kilometer point 44,7 are shown in Fig. 4:

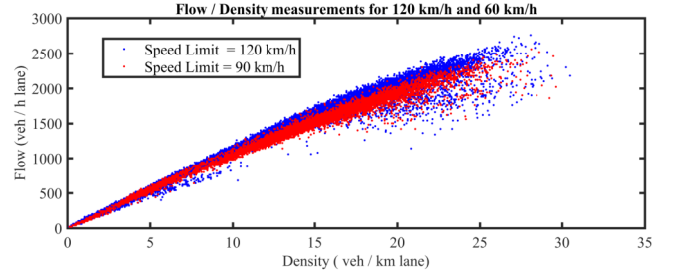


Fig. 4. Traffic data for Link 15 with $V_{c,i}(k) = 120$ km/h (blue) and $V_{c,i}(k) = 90$ km/h (red).

By inspection of the data, it can be seen that for both 90 km/h and 120 km/h there are some points that can be used to estimate capacity (because the slopes of the point clouds are strongly decreasing around densities of 27 or 28 veh/(km lane) and because the flows corresponding to higher densities are smaller than the capacity and have sparser distributions). Moreover, it can be seen that for $V_{c,i}(k) = 120$ km/h there are many points with higher flows (around the critical density) than the flows obtained with $V_{c,i}(k) = 90$ km/h (also around critical density).

If the point clouds are approximated, using the no-VSL desired speed equation (1), the FD capacity is reduced from 2418.2 veh/(h lane) (with $V_{c,i}(k) = 120$ km/h) to 2290 veh/(h lane) (with $V_{c,i}(k) = 90$ km/h). Even through this capacity reduction is relatively low (around 5%), it is expected that the reduction will be higher for lower speed limits (as predicted by the FDs in Fig. 3). The critical density increase is even smaller (around 4%) but, again, higher critical densities are expected for lower speed limits as shown in previous references.

Finally, and based on the FD parameters (a_i , $\rho_{c,i}$, and $v_{f,i}$) calibrated for the data measured with $V_{c,i}(k) = 120$ km/h, the parameters of the three VSL models previously presented (α_i , A_i , and E_i) are manually calibrated in

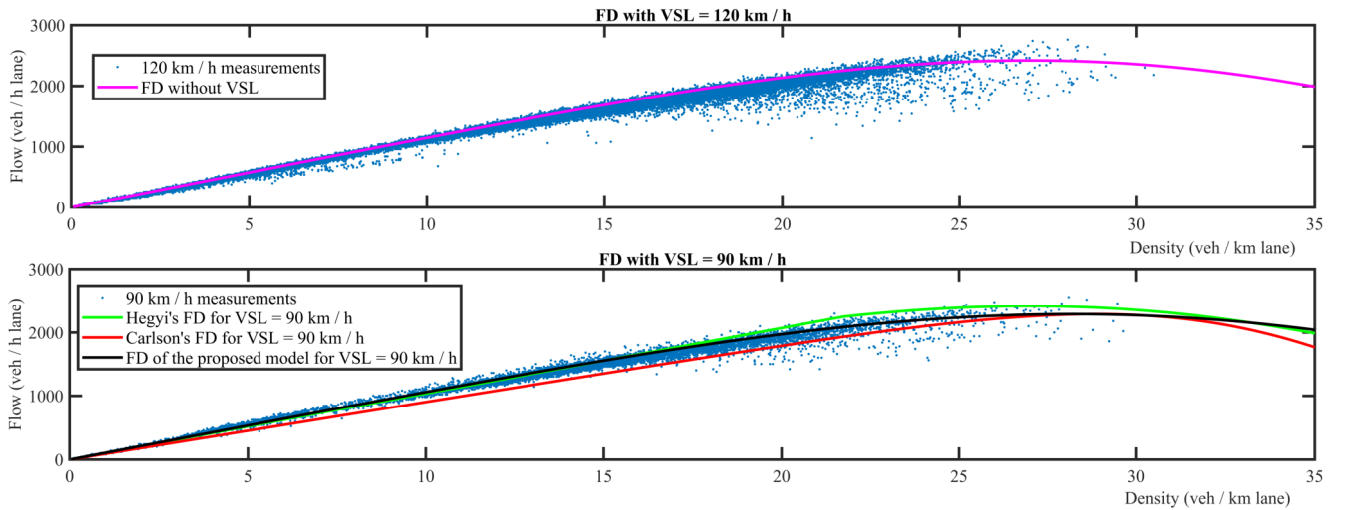


Fig. 5. Estimated FDs with $v_{f,i} = 115$ km/h, $\rho_{c,i} = 27$ veh/(km lane), $a_i = 4$, $V_{c,i}(k) = 90$ km/h, $\alpha_i = 0.15$ (for Hegyi's model), $A_i = 0.4245$ and $E_i = 5.5$ (for Carlson's model), and $\alpha_i = 0.18$, $A_i = 0.388$ and $E_i = 0.4$ (for the proposed model).

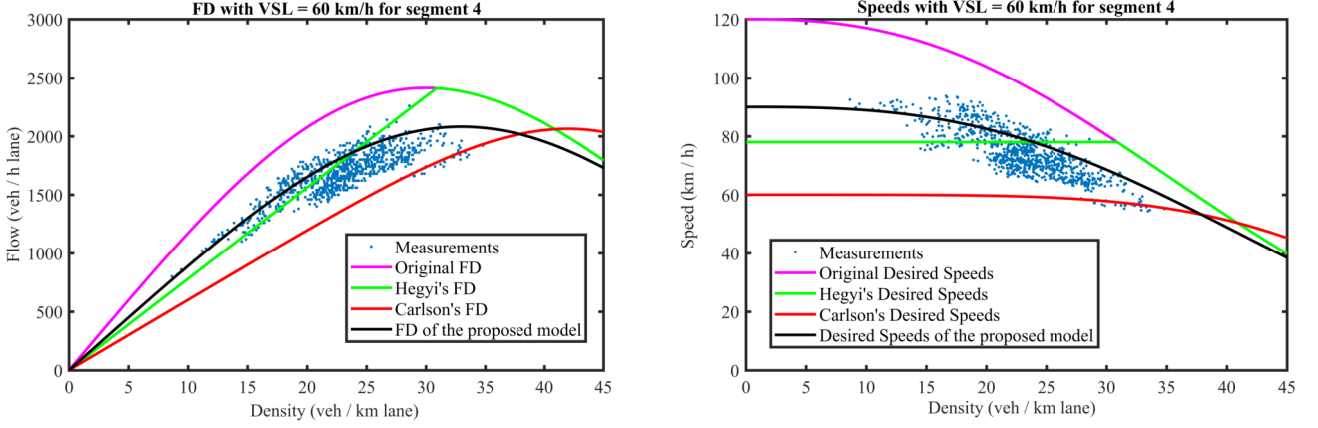


Fig. 6. FD diagrams and Desired Speeds for Link 4 with $v_{f,i} = 120$ km/h, $\rho_{c,i} = 30$ veh/(km lane), $a_i = 2.5$, $\alpha_i = 0.3$ (for Hegyi's model), $E_i = 3$, $A_i = 0.8$ (for Carlson's model), $\alpha_i = 0.5$, $E_i = 1.5$, and $A_i = 0.4$ (for the proposed model)

order to approximate the measurements obtained with $V_{c,i}(k) = 90$ km/h (see Fig. 5). Hegyi's model cannot be calibrated for capacity reductions different from the ones resulting from the model. Moreover, for high speed limits (like 90 km/h) the capacity is not reduced at all. In fact, the capacity and critical density given by Hegyi's model with $V_{c,i}(k) = 90$ are the same as for the original FD. On the other hand, Carlson's model and the proposed model are able to exactly match the new capacity of 2290 veh/(h lane).

According to the results in this section, the following conclusions may be drawn:

- As stated in previous references, the critical density of a link is increased by decreasing the corresponding VSL. This allows to store more vehicles (without reaching congestion) in a link upstream of a traffic jam and, therefore, to apply a VSL control algorithm for traffic efficiency improvement.
- At least for some scenarios, the capacity of a link is decreased due to a reduction of the corresponding VSL. This allows to reduce the flow in a link upstream of a traffic jam and, therefore, to increase the benefits of the previously cited VSL control algorithms for traffic efficiency improvement.
- The lack of capability of Hegyi's model to be calibrated for different VSL-induced capacities and critical densities may be one of its main drawbacks.
- On the other hand, Carlson's model and the proposed model may be adapted to match any VSL-induced critical density and capacity for each link. Therefore, they may deliver a better prediction for scenarios where VSLs are applied to links reaching capacity.

6. MODELING THE EFFECTS OF VSLs ON UNCONGESTED TRAFFIC

For freeway traffic control using VSLs, the speed limits are generally applied on uncongested links upstream of a bottleneck that has reached (or is going to reach soon) the critical density (Carlson et al. (2011); Hegyi and Hoogenboom (2010); Frejo et al. (2014)). Moreover, when a link is already congested due to a downstream bottleneck, the potential improvement in the traffic performance if a VSL

is applied on that link is not significant. Therefore, when using macroscopic models in order to compute or test VSL control algorithms, the most important aspect is how the speed of an uncongested link evolves when a speed limit is applied.

This section analyzes how each considered model predicts the response of the traffic when speed limits are applied on uncongested links and how these predictions match with the real data available from the A12 freeway.

Fig. 6 shows the flow and speed measurements for one link obtained when a speed limit of 60 km/h is applied. In this figure, data were used from all the days for which SPECIALIST was applied. The measurements obtained during the first two minutes after the application of the 60 km/h speed limit have been removed in order to remove transient effects of the application of the VSL. Moreover, the desired speeds and FDs obtained using each model are also shown in the figure.

For other links, the shapes of the measurements clouds are similar. Analyzing the measurements, the following conclusions can be drawn:

- For low densities, the compliance is very low. For example, it can be seen that the measured speeds are around 90 km/h when the densities are lower than 15 veh/km lane. However, it has to be pointed out that, based in the results of previous studies, the compliance may be increased by an enforcement measure. Nevertheless, even with an enforcement measure, only in a few scenarios the real traffic is going to have full compliance. Therefore, it can be concluded that including the driver compliance in a VSL model is quite beneficial in order to increase model accuracy.
- In order to properly model the VSL-induced free-flow speed, a high value of α_i (around 0.5 for A12 data) has to be used in some scenarios. Carlson's model considers that the VSL-induced free-flow speed is equal to the actual VSL so this model is not able to properly model this kind of response with such a low compliance.
- When densities are increased (even if the density is still below the critical density), the desired speeds

start to decrease substantially. For example, it can be seen in the figure that the measured speeds are around 70 km/h when the densities are about 25 veh/(km lane) (20 km/h less than the speed observed at low densities). Therefore, it can be concluded that the VSL-induced FD is not triangular for the uncongested part (equivalently to the FD without VSLs) and the use of an exponential equation increases the accuracy of the prediction.

- Carlson's model and the proposed model predict that the VSL-induced speeds decrease when densities increase because they use an exponential equation for the FD. Therefore, they are able to accurately approximate the behavior observed in the A12 data. On the other hand, using Hegyi's model the VSL-induced speeds are expected to have the same value for any density until they reach the speed given by the original FD (without VSLs). As a result, after calibration, the parameter α_i will probably take an intermediate value that predicts speeds lower than the real ones for low densities, but higher than the real ones for high, but uncongested, densities.

Combining the conclusions stated above and analyzing the figure, it can be seen that the proposed model is able to capture more accurately, and in a more general way, the effects of VSLs on uncongested links. This will allow to have a better, and more realistic, performance if a model-based VSL control algorithm is applied to an uncongested link in order to reduce the flow entering a traffic jam located downstream of the link where the VSL is applied.

7. CONCLUSIONS

This paper has analyzed the effects of VSLs on freeway traffic flow based on the field data available from the A12 freeway in The Netherlands. The following effects have been observed:

- The critical density of a link is increased if the corresponding VSL is decreased.
- The capacity of a link is reduced if the corresponding VSL is decreased.
- For low densities, the compliance can be very low.
- When densities are increased, the VSL-induced speeds start to decrease substantially.

Previously proposed VSL models are not able to be adapted in order to match all the previously mentioned effects. Hegyi's model can be adapted for different VSL-induced free-flow speeds, but it lacks the capability of modeling different capacity reductions and critical densities due to the effects of VSLs. On the other hand, Carlson's model is able to model different VSL-induced capacities and critical densities, but it does not consider different levels of compliance.

Therefore, a new VSL model has been presented combining characteristics of previously proposed models. The new model is able to reflect all the previously mentioned effects of VSLs on freeway traffic flow. In a future work, the considered models for VSL will be calibrated and validated using the same data set.

REFERENCES

- Carlson, R.C., Papamichail, I., and Papageorgiou, M. (2011). Local feedback-based Mainstream Traffic Flow Control on motorways using Variable Speed Limits. *IEEE Transactions on Intelligent Transportation Systems*, 12(4), 1261–1276.
- Carlson, R.C., Papamichail, I., Papageorgiou, M., and Messmer, A. (2010). Optimal motorway traffic flow control involving variable speed limits and ramp metering. *Transportation Science*, 44(2), 238–253.
- Daganzo, C.F. (1994). The Cell Transmission Model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological*, 28(4), 269–287.
- Frejo, J.R.D. and Camacho, E.F. (2012). Global versus local MPC algorithms in freeway traffic control with ramp metering and variable speed limits. *IEEE Transactions on Intelligent Transportation Systems*, 13(4), 1556–1565.
- Frejo, J.R.D., Núñez, A., De Schutter, B., and Camacho, E.F. (2014). Hybrid Model Predictive Control for freeway traffic using discrete speed limit signals. *Transportation Research Part C: Emerging Technologies*, 46, 309–325.
- Hegyi, A., De Schutter, B., and Hellendoorn, H. (2004). Optimal coordination of variable speed limits to suppress shock waves. *Transportation Research Record: Journal of the Transportation Research Board*, 1852, 167–174.
- Hegyi, A., De Schutter, B., and Hellendoorn, H. (2005). Model Predictive Control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C: Emerging Technologies*, 13, 185–209.
- Hegyi, A. and Hoogendoorn, S.P. (2010). Dynamic speed limit control to resolve shock waves on freeways - field test results of the SPECIALIST algorithm. In *13th International IEEE Conference on Intelligent Transportation Systems*, 519–524. Funchal, Portugal.
- Papageorgiou, M., Kosmatopoulos, E., and Papamichail, I. (2008). Effects of variable speed limits on motorway traffic flow. *Transportation Research Record: Journal of the Transportation Research Board*, 2047, 37–48.
- Papageorgiou, M., Papamichail, I., Messmer, A., and Yibing, W. (2010). *Fundamentals of Traffic Simulation: Traffic Simulation with METANET*. Springer, London.
- Papamichail, I., Kampitaki, K., Papageorgiou, M., and Messmer, A. (2008). Integrated ramp metering and variable speed limit control of motorway traffic flow. In *17th IFAC World Congress*, volume 41(2), 14084–14089.
- Soriguera, F., Martínez, I., Sala, M., and Menéndez, M. (2016). Effects of dynamic speed limits on a Dutch freeway. *Transportation Research Record: Journal of the Transportation Research Board*, 2560, 87–96.
- Soriguera, F., Martínez, I., Sala, M., and Menéndez, M. (2017). Effects of low speed limits on freeway traffic flow. *Transportation Research Part C: Emerging Technologies*, 77, 257–274.
- Van den Hoogen, E. and Smulders, S. (1994). Control by variable speed signs: results of the Dutch experiment. In *Seventh International Conference on Road Traffic Monitoring and Control*, 145–149. London, UK.