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Efficient model predictive control for variable speed limits by optimizing parameterized control schemes

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Abstract—This paper proposes an efficient model predictive control strategy that is based on the parameterization of a variable speed limit control scheme. Due to the parameterization, the solution spaces reduces, which leads to an improved computation time. The parameterized control scheme consists of a speed-limited area in which a constant speed limit is imposed. By changing the position of the head and tail of this speed-limited area over time it is possible to change the density and flow in and downstream of this area. The controller optimizes the location of the head and tail of this area over time in such a way that the flow into a bottleneck or jam wave is changed such that congestion can be prevented or resolved. An advantage of this approach is that the complexity of the optimization problem does not increase with an increase in the number of variable speed limit gantries. The controller is tested using a second-order macroscopic traffic flow model. It is shown that the controller improves the total time spent by all the vehicles in the network with 3.7% compared to the no-control situation. This improvement is realized by resolving a jam wave. It is also shown that the controller can achieve a better performance than other model predictive control strategies, when using the same amount of computation time.

I. INTRODUCTION

This paper addresses the challenge of developing an efficient – in terms of computational complexity – model predictive control (MPC) strategy for the optimization of variable speed limits (VSLs). The main task of the controller is to improve freeway throughput by reducing the amount of congestion. The reason why the throughput improves when reducing congestion is that the impact of the capacity drop is reduced. The capacity drop is the phenomenon that the flow downstream of congestion is lower than the free-flow capacity. Hence, by eliminating congestion the freeway flow can be increased and the throughput improved.

The approach proposed in this paper is based on the idea that the freeway flow can be reduced by means of VSLs. Several control strategies can be found in the literature that are of the flow-limiting type. The SPECIALIST algorithm imposes a speed-limited area upstream of a jam wave [1] of which the position of the head and tail of the area can vary over time. This speed-limited area has two functionalities. On the one hand it is used to reduce the flow into the jam wave such that it can resolve. On the other hand it stabilizes traffic such that new breakdowns are prevented. This algorithm has been applied in practice and successfully improved freeway throughput by resolving jam waves. Another approach is the local feedback based mainstream traffic flow controller which

changes the speed limit at a fixed distance upstream of a bottleneck [2]. In this way the flow into a bottleneck can be controlled such that it remains at or below capacity. Due to this, congestion is prevented at the bottleneck location, which improved the total travel time in a simulation environment. Another control strategy that can be used to resolve jam waves and to prevent congestion at bottlenecks is MPC [2], [3]. MPC has a number of desirable properties, for instance, it is able to explicitly take constraints into account, it can deal with various network structures, it can be extended to deal with multiple actuators, and it explicitly optimizes performance. However, the computation time is usually longer than the sampling time which is an important challenge of MPC.

A number of techniques to reduce the computation time of MPC strategies that are applied to traffic control problems exist. One approach is to simplify the model that is used for predictions [4], [5]. In this way a trade-off between computation time and accuracy can be made. Another approach is to distribute the optimization problem [6]. By means of distribution a similar trade-off can be made as well. A third approach to reduce the computation time is to parameterize an existing control approach [7] such that the number of variables that have to be optimized reduces. In their approach, every segment is controlled by the same feedback control law. At every time-step the three parameters – which are the same for every segment – of the feedback control law are optimized.

In this paper the computation time of a MPC strategy will be improved by parameterization of a control scheme. Similarly to the SPECIALIST approach, the control scheme consists of a speed-limited area in which a constant variable speed limit is imposed. However, by dynamically controlling the head and tail of this area over time, the flow and speed in and downstream of the control scheme can be adjusted. The controller that is developed in this paper optimizes the location of the head and tail of this area over time such that the flow into a jam wave or a bottleneck is controlled in such a way that a jam wave is resolved or congestion is prevented. In doing so, the complexity of the optimization problem does not increase with an increase in the number of variable message signs on the freeway.

II. CONSIDERATIONS FOR IMPLEMENTING VARIABLE SPEED LIMITS

As indicated in [8], a speed-limited area – as shown in Fig. 1 A – can be created by imposing VSLs. This area

can be used to buffer traffic for a while such that excess traffic demand is spread out. This mechanism can be used to prevent or resolve congestion. SPECIALIST is an example of an algorithm that uses a speed-limited area to resolve a jam wave [1]. In this paper, the speed-limited area has to satisfy the following properties:

- Only one speed-limited area can be imposed at a time;
- Only one value of the speed limits is used for control, implying that a VSL can either be on or off. The value of the VSL should be as low as possible such that congestion is not created by the VSLs;
- The speed in the speed-limited area is equal to the effective speed v^{eff} (km/h) corresponding to the imposed VSLs. The effective speed is defined as the speed with which vehicles drive in the speed-limited area which includes possible non-compliance. This can be estimated e.g. from field-tests as presented in [1];
- The dynamics of the head and tail of the speed-limited area should be such that the individual vehicles can only enter and exit the speed-limited area once. If an individual vehicle observes multiple fluctuations of the speed limits, this can lead to unsafe situations, annoyance or poor compliance. An example of a vehicle experiencing such fluctuations is shown in Fig. 1 A. In order to prevent such behavior, the following two conditions with respect to the head and tail of the speed-limited area are introduced;
- The positions $x^{\text{H,sl}}$ (km) and $x^{\text{T,sl}}$ (km) of respectively the head and the tail of the speed-limited area are allowed to propagate in the downstream direction with a speed that is lower or equal to the effective speed v^{eff} . In the upstream direction they can propagate with any speed.

An example of preventing congestion at a bottleneck by means of a speed-limited area is shown in Fig. 1 B. The top figure represents a time-space plot of a freeway with a bottleneck at location x_b (km). The bottom figure illustrates the demand entering the freeway at location x_0 (km). Assume that at time t_1 (h) an excess demand enters the freeway at location x_0 (km) and this demand reaches the bottleneck location x_b (km) at time t_2 (h). Congestion would appear when no control would be imposed. However, congestion may be prevented when imposing a speed-limited area starting from time t_2 .

The shape of the speed-limited area should be chosen such that the outflow from the speed-limited area into the bottleneck is equal to or below the capacity of the bottleneck. The shape of this area can be adjusted by changing the position of the head and tail over time. It follows from shock-wave theory that moving the position of the head of the speed-limited area in the upstream (downstream) direction results in an increase (decrease) of the density and flow downstream of the speed-limited area [1], [9]. Moving the tail results in a similar change of the density and flow in the speed-limited area.

III. FORMULATION OF THE SPEED-LIMITED AREA IN AN MPC FRAMEWORK

This section details the approach to formulate the speed-limited area in a MPC framework. MPC is a control strategy

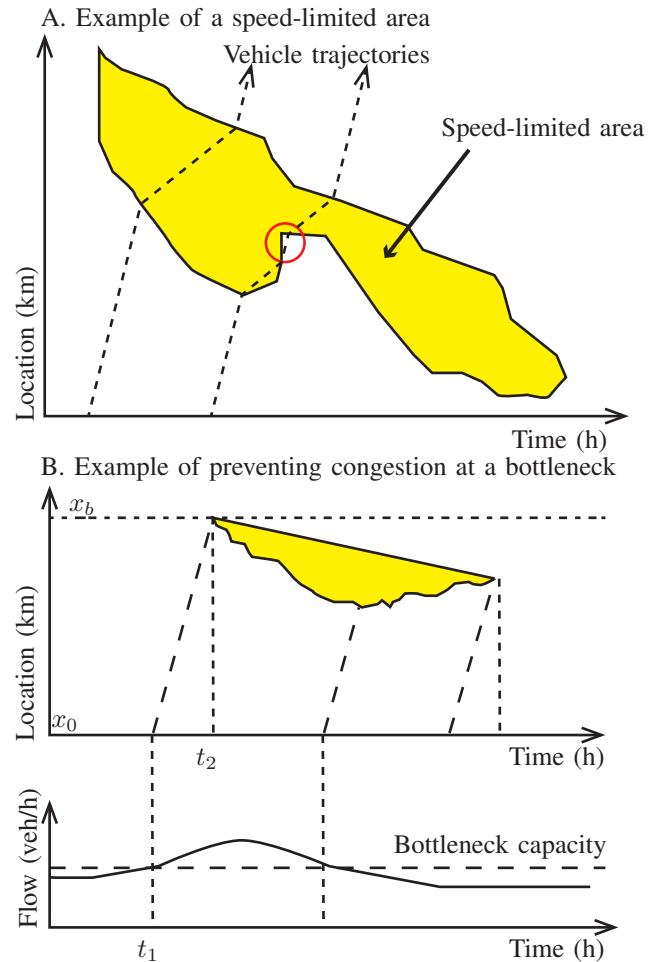


Fig. 1. A: Example of a speed-limited area that can be used to influence the traffic. Examples of vehicle trajectories are shown in the figure. In addition, the right vehicle trajectory illustrates that a vehicle experiences a speed limit drop twice, which should not occur. B: Top figure: example of a speed-limited area which can be used to prevent congestion at the bottleneck location x_b . Bottom figure: the demand entering the freeway at location x_0 .

in which a traffic flow model is used to predict the evolution of the traffic over a time horizon given the current traffic state, (future) disturbances, and various control inputs. The performance of the traffic flow process over this prediction horizon is determined using an objective function and the control input which provides the best performance is implemented. Only the first sample of the control input is imposed to the system, at the next time step new measurements become available and the optimization is repeated.

Before continuing, the timing of the approach will be introduced. A discrete-time second-order traffic model, METANET, is used to describe the evolution of the traffic [10]. A time step of the model is indicated with k (-) and the corresponding sampling time with T (h). The time step k refers to the period $t \in [Tk, T(k+1))$ (h). Similarly, a time-step of the controller is indicated with k^c (-) and the corresponding sampling time with T^c (h). The time step k^c refers to the period $t \in [T^c k^c, T^c(k^c+1))$ (h). The controller time steps are related to the model time steps via the ratio $C \in \mathbb{N}^+$ (-) as follows $k^c = \lfloor k/C \rfloor$, where the mathematical operator

$$q^{\text{lim},\mu,1}(k) = \begin{cases} \lambda_\mu v^{\text{lim},\mu,1}(k) \rho_\mu^{\text{crit}} \left[-a_\mu \ln \left(\frac{v^{\text{lim},\mu,1}(k)}{v_\mu^{\text{free}}} \right) \right]^{-1/a_\mu}, & \text{if } v^{\text{lim},\mu,1}(k) < V(\rho_\mu^{\text{crit}}) \\ \lambda_\mu V(\rho_\mu^{\text{crit}}) \rho_\mu^{\text{crit}}, & \text{if } v^{\text{lim},\mu,1}(k) \geq V(\rho_\mu^{\text{crit}}) \end{cases} \quad (5)$$

$\lfloor \cdot \rfloor$ means rounding to the nearest integer that is equal to or smaller than the argument of the function. It should be noted that at time step k^c the controller computes the control input for the next time step $k^c + 1$, thus the control input $u(k^c)$ cannot be adjusted by the controller. The controller predicts the evolution of the traffic from time step k^c until time step $k^c + N^p$ where N^p (-) is the prediction horizon. The control input from time $k^c + 1$ until time $k^c + N^c$ is optimized by the controller where N^c (-) is the control horizon and $N^c \leq N^p$.

A. The traffic flow model

The METANET model is adopted to predict the evolution of the traffic in the MPC. This section will first detail the original METANET model as proposed by [10]. Then the extensions proposed by [3] to include VSLs in the model are detailed, and finally a small extension which has been made to accommodate that the head and tail of the speed-limited area can lie anywhere within a segment. The METANET model is adopted, since, it provides a detailed description of the traffic dynamics and it can reproduce relevant traffic characteristics such as jam waves and the capacity drop at least qualitatively.

1) *The original METANET model:* In the METANET model, a freeway is divided into links m which are connected by nodes [10]. Each link m consists of N_m (-) segments of length L_m (km) with a number of λ_m (-) lanes. The flow $q_{m,i}(k)$ (veh/h), density $\rho_{m,i}(k)$ (veh/km/lane) and speed $v_{m,i}(k)$ (km/h) in a link are updated according to:

$$\begin{aligned} q_{m,i}(k) &= \rho_{m,i}(k) v_{m,i}(k) \lambda_m, \quad (1) \\ \rho_{m,i}(k+1) &= \rho_{m,i}(k) + \frac{T}{L_m \lambda_m} (q_{m,i-1}(k) - q_{m,i}(k)), \quad (2) \\ v_{m,i}(k+1) &= v_{m,i}(k) + \frac{T}{\tau} (v^{\text{FD}}(\rho_{m,i}(k)) - v_{m,i}(k)) \\ &\quad + \frac{T}{L_m} v_{m,i}(k) (v_{m,i-1}(k) - v_{m,i}(k)) \\ &\quad - \frac{\eta T}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa}, \quad (3) \end{aligned}$$

In the latter equation, τ , η , and κ are model parameters, and $v^{\text{FD}}(\rho_{m,i}(k))$ (km/h) is the equilibrium speed given by the fundamental diagram:

$$v^{\text{FD}}(\rho_{m,i}(k)) = v_m^{\text{free}} \exp \left(-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_m^{\text{crit}}(k)} \right)^{a_m} \right), \quad (4)$$

where a_m (-) is a model parameter, the speed v_m^{free} (km/h) is the free-flow speed of the freeway, and the density ρ_m^{crit} (veh/km) is the critical density at which traffic becomes unstable.

An origin is modeled using a simple queuing model describing the number of vehicles $w_0(k)$ (veh) in the origin queue as a function of the demand $d_0(k)$ (veh) and the out-flow

$q_0(k)$:

$$w_0(k+1) = w_0(k) + T(d_0(k) - q_0(k)), \quad (6)$$

$$q_0(k) = \min \left[d_0(k) + \frac{w_0(k)}{T}, Q_0 \frac{\rho_m^{\text{max}} - \rho_{m,1}(k)}{\rho_m^{\text{max}} - \rho_m^{\text{crit}}} \right], \quad (7)$$

with Q_0 (veh/h) the on-ramp capacity.

2) *Extension with variable speed limits:* The following adjustments to the original model are implemented in order to model the effect of VSLs [3]. First of all, the parameter η in (3) is replaced by the parameter $\eta_{m,i}(k)$ which can take the value η^{high} when the downstream density is higher than the density $\rho_{m,i+1}(k)$ in segment i , and it can take the value η^{low} when the downstream density is lower.

Secondly, equation (7) which updates the origin outflow is replaced by the following equation:

$$q_0(k) = \min \left[d_0(k) + \frac{w_0(k)}{T}, q^{\text{lim},\mu,1}(k) \right], \quad (8)$$

where the flow $q^{\text{lim},\mu,1}(k)$, see (5), is determined by the traffic condition in the first link and the speed $v^{\text{lim},\mu,1}(k) = \min[v_{m,1}^{\text{ctrl}}(k), v_{\mu 1}(k)]$.

Thirdly, the equilibrium speed $v^{\text{FD}}(\rho_{m,i}(k))$ in (3) is replaced by the speed $V(\rho_{m,i}(k))$ (km/h) given by:

$$V(\rho_{m,i}(k)) = \min [v^{\text{FD}}(\rho_{m,i}(k)), v_{m,i}^{\text{ctrl}}(k)]. \quad (9)$$

Finally, when a link i_{last} has no leaving link – i.e. it is the most downstream link – the downstream density $\rho_{m,i_{\text{last}}+1}$ is equal to:

$$\rho_{m,i_{\text{last}}+1} = \max [\rho^{\text{DS}}(k), \min[\rho_{m,i_{\text{last}}}(k), \rho_m^{\text{crit}}]], \quad (10)$$

where the density $\rho^{\text{DS}}(k)$ (veh/km/lane) is the destination density which can be used as a boundary condition to the model.

3) *Extension with a speed-limited area:* In this paper, the VSLs $v_{m,i}^{\text{ctrl}}(k)$ are determined by the head $x^{\text{H,sl}}(k)$ (km) and tail $x^{\text{T,sl}}(k)$ (km) of the speed-limited area as follows:

$$v_{m,i}^{\text{ctrl}}(k) = \begin{cases} v^{\text{eff}} & \text{if } x^{\text{H,sl}}(k) > x_{m,i} \wedge \dots \\ & x^{\text{T,sl}}(k) < x_{m,i} + L_{m,i} \wedge \dots \\ v^{\text{free}} & \text{otherwise,} \end{cases} \quad (11)$$

where x_i (km) is the most upstream location of the segment i , and L_i (km) is the length of segment i .

The position of the head and tail of the speed-limited area can lie anywhere within a segment. However, in the methods mentioned so far, the speed-limited area can either cover an entire segment or not cover it at all. This implies that the gradient of the objective function – as detailed in the next section – is discontinuous. In order to realize a

$$\gamma_{m,i}(k^c) = \max \left[\frac{L_{m,i} - \max[x^{\text{T,sl}}(k^c) - x_{m,i}, 0] - \max[x_{m,i} + L_{m,i} - x^{\text{H,sl}}(k^c), 0]}{L_{m,i}}, 0 \right] \quad (12)$$

continuous gradient – which is required when using gradient-based optimization techniques, such as, sequential quadratic programming – a parameter $\gamma_{m,i}(k)$ (-) is introduced. The parameter $\gamma_{m,i}(k)$ denotes the fraction of the segment that is covered by speed limits as defined in (12). This factor is used to compute the speed $v_{m,i}^{\text{ctrl}}(k)$ in the segment by taking the weighted average of the effective speed v^{eff} and the equilibrium speed $v^{\text{FD}}(\rho_{m,i}(k))$:

$$v_{m,i}^{\text{ctrl}}(k) = \gamma_{m,i}(k)v^{\text{eff}} + (1 - \gamma_{m,i}(k))v^{\text{FD}}(\rho_{m,i}(k)). \quad (13)$$

B. The objective function, input vector, and constraints

The objective of the controller is to minimize the Total Time Spent (TTS) by all the vehicles on the freeway by changing the VSLs over the time-horizon from $k^c + 1$ until $k^c + N^c$. The following objective function $J(k^c)$ expresses the TTS:

$$J(k^c) = T \sum_{k=k^c+1}^{k^c+N^c} \left\{ \sum_{(m,i) \in I^{\text{links}}} \rho_{m,i}(k) L_m \lambda_m + \sum_{o \in I^{\text{orig}}} w_o(k) \right\}. \quad (14)$$

Here, the set I^{links} (-) is the set of indexes of all pairs of segments and links, and the set I^{orig} (-) is the set of all origin indexes.

The values of the objective function can change when the value of the VSLs $v_{m,i}^{\text{ctrl}}(k^c + \tilde{k})$ are adjusted, where \tilde{k} is in the range $\{1, \dots, N^c\}$. The value of the VSLs can be adjusted by changing the position of the head $x^{\text{H,sl}}(k^c + \tilde{k})$ and tail $x^{\text{T,sl}}(k^c + \tilde{k})$ of the speed-limited area.

The evolution of the head and tail of the speed-limited area is described by the initial location of the head $x^{\text{H,sl}}(k^c + 1)$ (km) and tail $x^{\text{T,sl}}(k^c + 1)$ (km), and the speed $v^{\text{H,sl}}(k^c + \tilde{k})$ (km/h) and $v^{\text{T,sl}}(k^c + \tilde{k})$ (km/h) of the head and tail over time respectively. Thus, the control vector $\bar{u}(k^c)$ has the following form:

$$\bar{u}(k^c) = \begin{bmatrix} x^{\text{H,sl}}(k^c + 1|k^c) \\ x^{\text{T,sl}}(k^c + 1|k^c) \\ v^{\text{H,sl}}(k^c + 1|k^c) \\ \dots \\ v^{\text{H,sl}}(k^c + N^c|k^c) \\ v^{\text{T,sl}}(k^c + 1|k^c) \\ \dots \\ v^{\text{T,sl}}(k^c + N^c|k^c) \end{bmatrix}. \quad (15)$$

After the control horizon N^c , until the prediction horizon N^p , the speed of the head and tail locations are assumed to remain constant:

$$v^{\text{H,sl}}(k^c + \tilde{k}|k^c) = v^{\text{H,sl}}(k^c + N^c|k^c) \text{ if } \tilde{k} > N^c, \quad (16)$$

$$v^{\text{T,sl}}(k^c + \tilde{k}|k^c) = v^{\text{T,sl}}(k^c + N^c|k^c) \text{ if } \tilde{k} > N^c. \quad (17)$$

Based on the control vector, the location of the head and the tail of the control scheme can be computed:

$$x^{\text{H,sl}}(k^c + \tilde{k}|k^c) = x^{\text{H,sl}}(k^c + 1) + \sum_{j=k^c+1}^{\tilde{k}} v^{\text{H,sl}}(j)T^c, \quad (18)$$

$$x^{\text{T,sl}}(k^c + \tilde{k}|k^c) = x^{\text{T,sl}}(k^c + 1) + \sum_{j=k^c+1}^{\tilde{k}} v^{\text{T,sl}}(j)T^c. \quad (19)$$

The following constraints have to be respected in the optimization problem:

- The position of the head and tail have to lie within the upstream bounds $x^{\text{H},0}$ (km) and $x^{\text{H},\text{end}}$ (km) and downstream bounds $x^{\text{H},\text{end}}$ (km) and $x^{\text{H},0}$ (km):

$$x^{\text{H},0} \leq x^{\text{H,sl}}(k^c + 1) \leq x^{\text{H},\text{end}}, \quad (20)$$

$$x^{\text{H},0} \leq x^{\text{T,sl}}(k^c + 1) \leq x^{\text{H},\text{end}}. \quad (21)$$

If at time step k^c the speed limits are not active, i.e. when $x^{\text{H,sl}}(k^c|k^c - 1) \leq x^{\text{T,sl}}(k^c|k^c - 1)$, then these bounds are equal to the upstream x_0 (km) and downstream end of the freeway x_{end} (km). However, when the speed limits are active at time k^c , then the location of the head $x^{\text{H,sl}}(k^c + 1|k^c)$ and tail $x^{\text{T,sl}}(k^c + 1|k^c)$ at time step $k^c + 1$ should be equal to the previously computed values $x^{\text{H,sl}}(k^c + 1|k^c - 1)$ and $x^{\text{T,sl}}(k^c + 1|k^c - 1)$. In that case, the constraints should be set to the following:

$$x^{\text{H},0} = x^{\text{H,sl}}(k^c + 1|k^c - 1), \quad (22)$$

$$x^{\text{H},\text{end}} = x^{\text{H,sl}}(k^c + 1|k^c - 1), \quad (23)$$

$$x^{\text{H},0} = x^{\text{T,sl}}(k^c + 1|k^c - 1), \quad (24)$$

$$x^{\text{H},\text{end}} = x^{\text{T,sl}}(k^c + 1|k^c - 1). \quad (25)$$

- The head and tail are allowed to propagate downstream with at most v^{eff} (km/h) or to propagate upstream with any speed:

$$v^{\text{H,sl}}(k^c + \hat{k}) \leq v^{\text{eff}}, \quad (26)$$

$$v^{\text{T,sl}}(k^c + \hat{k}) \leq v^{\text{eff}}, \quad (27)$$

where \hat{k} is in the range $\{1, \dots, N^c\}$.

- The initial position of the head should be equal to, or more downstream than the initial position of the tail:

$$x^{\text{H,sl}}(k^c + 1) \geq x^{\text{T,sl}}(k^c + 1). \quad (28)$$

To summarize, all of these constraints are given by:

$$\begin{bmatrix} -\infty \\ x^{\text{H},0} \\ x^{\text{H},0} \\ -\infty \\ \vdots \\ -\infty \end{bmatrix} \leq \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \bar{u}(k^c) \leq \begin{bmatrix} 0 \\ x^{\text{H},\text{end}} \\ x^{\text{H},\text{end}} \\ v^{\text{eff}} \\ \vdots \\ v^{\text{eff}} \end{bmatrix}. \quad (29)$$

Now, the MPC optimization problem can be formulated:

$$\min_{\bar{u}(k^c)} J(k^c)$$

$$\text{Subject to (1) – (13), (29),} \quad (30)$$

$$\rho_{m,i}(k^c), v_{m,i}^{\text{ctrl}}(k^c), \rho^{\text{DS}}(k^c + \hat{k}), d_0(k^c + \hat{k}), \quad (31)$$

$$\text{where } k \text{ in (1) – (13) refers to } k|k^c. \quad (32)$$

IV. EVALUATION

The objective of the evaluation is to assess whether the controller can improve the TTS by resolving a jam wave using VSLs, and to compare the trade-off between TTS and computation time of this approach with other approaches.

To this end, a 30 km long freeway without on-ramps and off-ramps is modeled using a process model that is almost the same as the prediction model. The only difference between the two models being that the parameter $\gamma_{m,i}(k)$ is set to 1 in the process model such that the entire segment is either speed limited or not.

The freeway consists of an origin and 30 identical segments with a length of 1 km and 2 lanes. Every segment has the same parameters, adopted from [10], namely: $T = 10\text{s}$, $\tau = 18\text{ s}$, $\kappa = 40$ (veh/km/lane), $\rho_{\text{crit}} = 33.5$ veh/km/lane, $a_m = 1.867$, $v_{\text{free}} = 102$ km/h, $\eta^{\text{high}} = 65$ km/h², $\eta^{\text{low}} = 30$ km/h². Using these parameters, a capacity of 2000 veh/h/lane is realized and a capacity drop can be observed. The freeway traffic is simulated for scenarios of 2 hours. All the segments are controlled by means of VSLs. The value of the effective speed limit v^{eff} is set to 50 km/h. The position of the head and tail of the speed-limited area is allowed to change every 60 seconds.

The evaluation is carried out using Matlab R2012b on a computer with a 3.6 GHz processor and 16 Gb RAM. For the optimization the Sequential Quadratic Programming algorithm of the SNOPT solver of the TOMLAB optimization toolbox is used, the major optimality tolerance of this approach is set to 0.02, the major iterations limit is set to 20.

A scenario in which a jam wave is present on the freeway is evaluated. Time-space diagrams of the no-control situation are presented in Fig. 3 (a)–(d). It can be observed that a jam wave enters the freeway at the most downstream end as shown in Fig. 3. This jam is created using the density profile of the downstream end of the freeway as shown in Fig. 3 (d). The demand-profile $d_0(t)$ is constant and equal to 1950 veh/h/lane. The capacity drop due to this jam wave is approximately 5%. The total time spent of this scenario is 3935 veh-h.

The controller is started at 1560 seconds, when the jam wave is fully formed, and it is expected to improve the TTS by resolving the jam wave. The controller is given a maximum of 2 minutes CPU time per sampling time step during which it can try out several starting points for the optimization. It must be noted that the CPU time required when the controller is started is approximately 90 seconds. During the other time steps, the CPU time required to optimize from a single starting point is approximately 10 seconds. The reason why the initial CPU time is longer is that the locations $x^{\text{H,sl}}(k^c + 1|k^c)$ and $x^{\text{T,sl}}(k^c + 1|k^c)$ have to be optimized when the controller is started. The initial computation time might be further reduced by providing a better starting point. However, this is not a trivial task and left for further research.

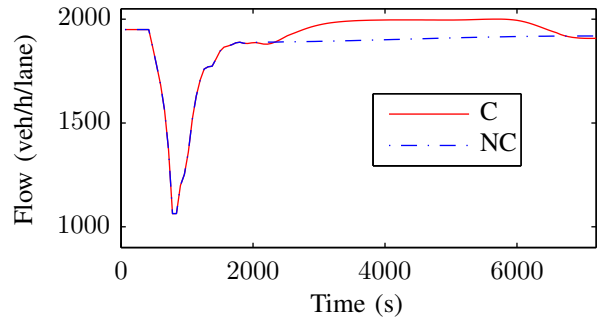


Fig. 2. Plot of the no-control (NC), and control (C) outflow

TABLE I. COMPARISON OF PERFORMANCE OF DIFFERENT APPROACHES FOR DIFFERENT AMOUNTS OF CPU TIME.

CPU time	5 min		10 min	
	TTS (veh-h)	% TTS gain	TTS (veh-h)	% TTS gain
Method				
No control	3935	-	3935	-
Parameterized control scheme	3797	3.5	3799	3.5
Hegyí <i>et al.</i> [3]	3924	0.3	3846	2.3
Zegeye <i>et al.</i> [7]	3922	0.3	3922	0.3

Time-space diagrams of the evaluation results are presented in Fig. 3 (e)–(h) and comparisons of the control and no-control outflow are shown in Fig. 2. This figure shows that by instantaneously imposing speed limits over a section of 10 km upstream of the jam wave at time 1560 s, the flow into the jam wave is reduced such that it resolves. After that, the speed-limited area is chosen in such a way that the freeway outflow is near the freeway capacity. This mechanism is similar to the way in which a speed-limited area is imposed by the SPECIALIST algorithm [1]. Due to the fact that the jam wave is resolved, the TTS is reduced to 3796 veh-h which implies a gain of 3.7 % when compared to the no-control situation.

The performance in terms of TTS improvement and CPU time used is compared with two other control strategies. These control strategies are: (1) the MPC strategy of [3] in which the value of the VSLs of every variable message sign are optimized, and (2) the parameterized MPC strategy of [7] in which the parameters of a feedback control law for VSLs are optimized. It must be noted that the method of [11] is used to implement the parameterization of [7]. In order to obtain a fair comparison, every control strategy receives the same amount of CPU time, namely 5 minutes and 10 minutes, per sampling time step.

Table I provides an overview of the percentage TTS improvement compared to the no-control situation. It can be observed that the approach proposed in this paper achieves the best performance gain for various amounts of computation times. It must be noted that in order to obtain these results, the major optimality tolerance of the optimization algorithm has been increased to 0.1. Also the major iterations limit was reduced to 3 for the approach of [3]. The reason why the approach of [3] has worse performance is that it needs more computation time to reach the optimum. The reason why the approach of [7] has lower performance is that it cannot apply a spatially heterogeneous control action.

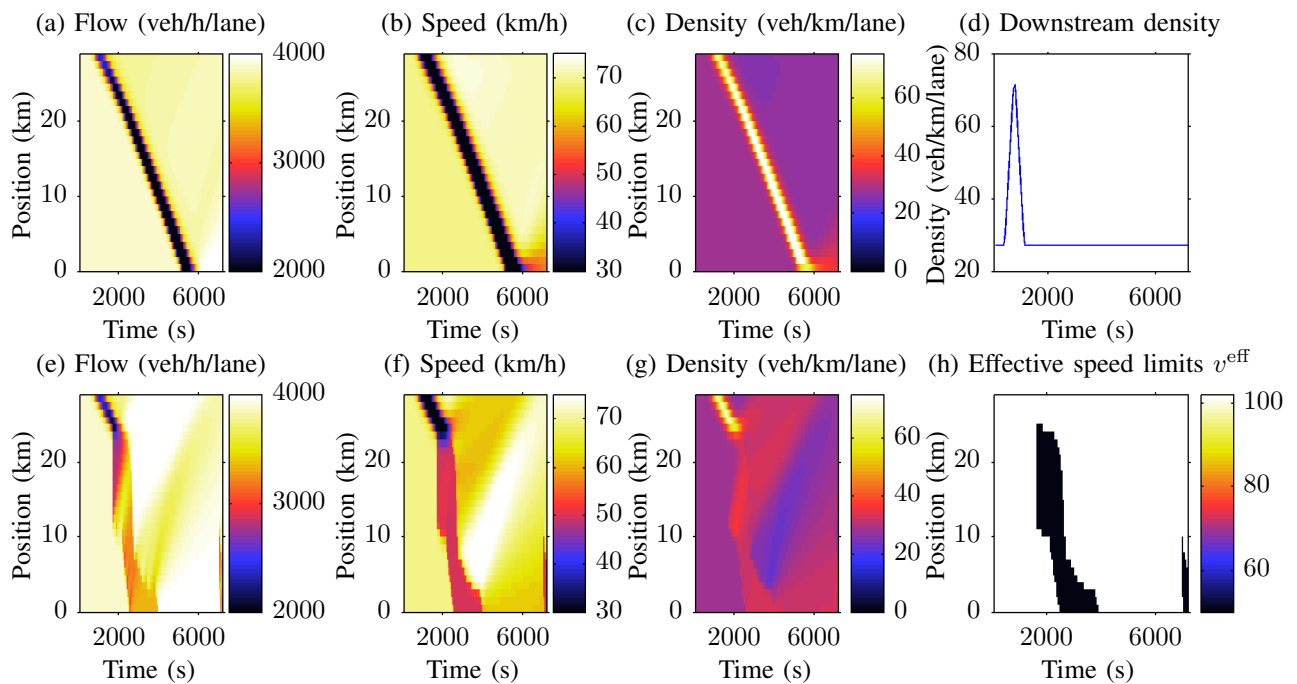


Fig. 3. Time-space diagrams of (a,e) the flow, (b,f) the speed, and (c,g) the density, where the driving direction of the traffic is from bottom to top. Plots (a)–(d) show the uncontrolled scenario. Plot (d) shows the density at the downstream end of the freeway. This downstream density represents a jam wave entering from the most downstream end and propagating in the upstream direction. Plot (h) shows the imposed VSLs. Plots (e)–(h) show the controlled situation. It can be observed that the jam wave is resolved by imposing a speed-limited area.

V. CONCLUSIONS AND RECOMMENDATIONS

In this paper the computation time of an MPC strategy for VSLs was improved by parameterizing a control scheme. The main advantages of this approach are that the controller can achieve a better performance than other MPC strategies, when using the same amount of computation time, and that opposed to some other approaches the computational complexity is not a function of the number of variable message signs. Evaluations were carried out which showed that the controller can improve the TTS with 3.7% compared to the no-control situation by resolving a jam wave.

Further research will aim at extending the controller to deal with ramp metering, to include gradual transitions of the speed limits, and applying the controller to larger freeway networks. Also, more extensive evaluations will be carried out in order to investigate the impact of the model-reality mismatch. The approach is suited to be applied to other traffic models as well which would further extend the applicability of this control method. The computation time could be further improved by providing a better guess for the starting point for the optimization.

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