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A Sequential Linear Programming Approach for Flow Assignment in Intermodal Freight Transport*

L. Li, R. R. Negenborn, and B. De Schutter

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Abstract— **Intermodal freight transport has been proposed as a potential solution for achieving efficient and sustainable transport systems in the hinterland of the Port of Rotterdam. The routing problem arising in container distribution and collection processes is an important issue and similarly the flow assignment problem arises at a more aggregate level. In this paper, we first briefly review the existing literature on routing for intermodal freight transport. Next, an extended intermodal transport network model is formulated for the time-varying and load-dependent travel time on roads. The optimal intermodal freight flow assignment problem becomes a nonlinear and non-convex optimization problem. A sequential linear programming (SLP) approach is proposed for determining the freight flow assignment. Simulation results show the effectiveness of the proposed SLP approach.**

I. INTRODUCTION

Increasing cargo throughputs have been forecasted by the Rotterdam Port authority for the coming twenty years under four different economic scenarios, e.g., from 430 million tonnes to 650 million tonnes in the European Trend scenario [1]. These upcoming throughputs will bring challenges for both the deep-see port and its hinterland transport networks. With the construction of Maasvlakte 2, Port of Rotterdam will undergo a large increase in the capacities of cargo handling and storage [1]–[3]. Therefore, more and more attention is being paid to improve the operational efficiency of transport systems that connect the deep-see port to the hinterland and that have been suffering from frequent road traffic congestion, traffic pollution, etc.

Intermodal freight transport has been proposed by the industry and academic researchers as a potential solution for achieving efficient and sustainable transport systems. Intermodal freight transport involves the movement of goods in one and the same loading unit by successive modes of transport without handling of the goods themselves when changing modes [4]. It integrates the use of different modes of transport (e.g., freeways, railways, waterways, etc.) to deliver cargo in a cost and time effective way while making appropriate use of the existing infrastructure. In particular, intermodal freight transport provides the possibility to increase the market share of trains and barges that are considered to be more environmentally friendly than trucks, by utilizing cargo transport and performing modal transfers in an appropriate manner during the freight delivery process. Planning problems in intermodal freight transport

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are categorized regarding their different time horizons into three different decision-making levels, i.e., the strategic level, the tactical level and the operational level. For a detailed overview of research work done in these three decision-making levels, the reader is referred to the literature reviews of Macharis and Bontekoning [5] and Caris, et al. [6]. In this paper, we investigate freight flow assignment problems at the operational level faced by intermodal freight forwarders.

The container is one prevailing form of a loading unit for freight transport in the modern logistics system. To make sure containers are transported efficiently through intermodal transport networks, appropriate routes need to be selected. However, it is too complicated to investigate each container's delivery process when studying a large-scale intermodal transport network. Therefore, we consider the movement of containers as a flow at a more aggregated level [7]. From the container flow perspective, one primary task in intermodal freight transport is to assign freight (container) flows from their origins to destinations over the network in an optimal way such that the total transport time and the total delivery cost are minimized. In this paper, we formulate this assignment task as a nonlinear and non-convex optimization problem using an extended intermodal transport network model. A sequential linear programming approach is proposed to solve the resulting optimization problem.

This paper has the following structure. Section II presents a brief literature review in the field of routing for intermodal freight transport. Section III starts with a short recapitulation of the generic intermodal transport network model developed in the authors' previous work and proposes a model extension to capture the time-varying and load-dependent travel time on roads. A sequential linear programming approach is proposed to determine the intermodal freight flow assignment in Section IV. In Section V, simulation results are presented for the implementation of the extended model and the SLP approach.

II. INTERMODAL FREIGHT ROUTING APPROACHES

In intermodal freight routing, an intermodal transport network can be modeled as a directed graph, in which nodes represent entities like deep-sea ports, inland ports, and terminals in the hinterland, while links represent entities like freeways, railways, waterways, and perhaps modality transfers. Therefore, routing choice approaches can be categorized on the basis of the parameter properties of the corresponding graph. Here, graph parameters refer to parameters like node handling rate/cost/capacity, link transport time/cost/capacity, modal transfer time/cost/capacity, etc.

In [8], static link transport time/cost and static modality transfer time/cost are studied. The K-shortest path algorithm is applied to determine the K-least expensive modality combinations for all origin-destination pairs in intermodal freight transport. Some work has also been done in intermodal or multimodal freight transport with different concerns (e.g., international intermodal transport networks [9], [10], carbon emissions [11], etc.) while considering static graph parameters.

A time-dependent case is considered by [12]. In this case, transport networks provide different modes of transport and are characterized by dynamic arc travel times and switching delays. A time-dependent intermodal least-time algorithm is developed based on the label correcting approach for transport operations in this time-dependent case. In [13], a parallel algorithm is presented to solve a time-dependent transport problem in transport networks by decomposing the whole network according to transport modes and regions.

All the above mentioned works are conducted for either deterministic link transport times and/or mode transfer times or time-dependent links transport times and/or mode transfer times that are considered as a given input of the routing algorithm. To the best of the authors' knowledge, there are no published works that study interactions between link transport times and freight flow assignments. Therefore, we extend the generic intermodal transport network model developed in our previous work so as to be able to capture the mutual influence mechanism between link transport times and freight flow assignments in the road network.

III. An extended intermodal transport network model

A generic intermodal transport network model has been proposed in our previous work [7] based on a perspective of container flows and the use of the multiple node method for modeling modality transfer phenomena inside intermodal terminals. In this section, we first give a brief recapitulation of the generic intermodal transport network model proposed in [7]. Next, a model extension for the road network is formulated in case of multi-class traffic flows on freeways.

A. A generic intermodal transport network model

An intermodal transport network can be represented as a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$. The node set $\mathcal{V} = \mathcal{V}_{\text{truck}} \cup \mathcal{V}_{\text{train}} \cup \mathcal{V}_{\text{barge}} \cup \mathcal{V}_{\text{store}}$ is a finite nonempty set with the sets $\mathcal{V}_{\text{truck}}$, $\mathcal{V}_{\text{train}}$, $\mathcal{V}_{\text{barge}}$, and $\mathcal{V}_{\text{store}}$ representing truck terminals, train terminals, barge terminals, and storage yards shared by different single-model terminals inside each intermodal terminal of the network, respectively. The set $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$ represents transport modes and mode transfer types in the network with $\mathcal{M}_1 = \{\text{truck}, \text{train}, \text{barge}, \text{store}\}$ and $\mathcal{M}_2 = \{m_1 \rightarrow m_2 | m_1, m_2 \in \mathcal{M}_1 \text{ and } m_1 \neq m_2\}$. The link set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \times \mathcal{M}$ represents all available connections among nodes. A link (i, j, m) with $i, j \in \mathcal{V}$ and $m \in \mathcal{M}$ will be denoted by $l_{i,j}^m$. According to whether a model transfer happens or not in one link, this link is categorized as transport link or transfer link.

Each transport demand (o, d) in the intermodal transport network belongs to the transport demand set $\mathcal{O}_{\text{od}} \subseteq \mathcal{V} \times \mathcal{V}$. For each pair $(o, d) \in \mathcal{O}_{\text{od}}$ we denote the volume of this transport demand as $d_{o,d}$. The generic intermodal transport network model is a discrete-time model with T_{s} (h) as the time step size. It is formulated as follows:

$$
\begin{aligned}
x_{i,o,d}(k+1) &= x_{i,o,d}(k) + \sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k) T_{\text{s}} \\
&\quad - \sum_{(j,m) \in \mathcal{N}_i^{\text{out}}} y_{i,j,o,d}^m(k) T_{\text{s}} + d_{i,o,d}^{\text{in}}(k) T_{\text{s}} \\
&\quad - d_{i,o,d}^{\text{out}}(k) T_{\text{s}}, \quad \forall (o,d) \in \mathcal{O}_{\text{od}}, \\
&\quad \forall i, j \in \mathcal{V}, \; \forall m \in \mathcal{M}, \; \forall k,
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
q_{i,j,o,d}^{m,\text{out}}(k) &= \sum_{\substack{k_e = k - t_{i,j}^{m,max} \\ k_e + t_{ij}^m(k_e) = k}}^{k-1} q_{i,j,o,d}^{m,in}(k_e), \; \forall k, \\
&\quad \forall (i,j,m) \in \mathcal{E}, \; \forall (o,d) \in \mathcal{O}_{\text{od}},
\end{aligned} \tag{2}
$$

$$
x_{i,j,o,d}^m(k+1) = x_{i,j,o,d}^m(k) + \left(q_{i,j,o,d}^{m,\text{in}}(k) - q_{i,j,o,d}^{m,\text{out}}(k) \right) T_{\text{s}},
$$
$$
\forall (i,j,m) \in \mathcal{E}, \; \forall (o,d) \in \mathcal{O}_{\text{od}}, \; \forall k, \tag{3}
$$

$$
\begin{aligned}
q_{i,j,o,d}^{m,\text{in}}(k) &= y_{i,j,o,d}^m(k) \quad \forall i \in \mathcal{V}, \forall (j,m) \in \mathcal{N}_i^{\text{out}} \\
&\quad \forall (o,d) \in \mathcal{O}_{\text{od}}, \forall k,
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
u_{i,j,o,d}^m(k) &= q_{i,j,o,d}^{m,\text{out}}(k) \quad \forall i \in \mathcal{V}, \forall (j,m) \in \mathcal{N}_i^{\text{in}}, \\
&\quad \forall (o,d) \in \mathcal{O}_{\text{od}}, \forall k,
\end{aligned} \tag{5}
$$

$$
\sum_{(o,d) \in \mathcal{O}_{\text{od}}} \sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k) \leq h_i^{\text{in}}, \forall i \in \mathcal{V}, \; \forall k, \tag{6}
$$

$$
\sum_{(o,d) \in \mathcal{O}_{\text{od}}} x_{i,o,d}(k) \leq S_i, \forall i \in \mathcal{V}, \; \forall k, \tag{7}
$$

$$
\sum_{(o,d) \in \mathcal{O}_{\text{od}}} \sum_{(j,m) \in \mathcal{N}_i^{\text{out}}} y_{i,j,o,d}^m(k) \leq h_i^{\text{out}}, \forall i \in \mathcal{V}, \; \forall k, \tag{8}
$$

$$
\sum_{(o,d) \in \mathcal{O}_{\text{od}}} x_{i,j,o,d}^m(k) \leq C_{i,j}^m, \forall (i,j,m) \in \mathcal{E}, \; \forall k, \tag{9}
$$

$$
\sum_{(o,d) \in \mathcal{O}_{\text{od}}} q_{i,j,o,d}^{m,\text{in}}(k) \leq C_{i,j}^{m,\text{in}}, \forall (i,j,m) \in \mathcal{E}, \; \forall k, \tag{10}
$$

where
- The value of $x_{i,o,d}(k)$ (TEUs) is the number of containers corresponding to transport demand (o, d) and staying at node i at time step k.
- The value of $u_{j,i,o,d}^m(k)$ (TEUs/h) is the container flow corresponding to transport demand (o, d) and entering node i through link $l_{j,i}^m, (j, m) \in \mathcal{N}_i^{\text{in}}$ at time step k. The set $\mathcal{N}_i^{\text{in}}$ is defined as

$$\mathcal{N}_i^{\text{in}} = \{(j,m) \mid l_{j,i}^m \text{ is an incoming link for node } i\}.$$

The value of $u_{j,i,o,d}^m(k)$ equals zero when $i = o$ (which implies that node i is actually the origin node o of transport demand (o, d)).

- The value of $y_{i,j,o,d}^m(k)$ (TEUs/h) is the container flow corresponding to transport demand (o,d) and leaving node i through link $l_{i,j}^m$, $(j,m) \in \mathcal{N}_i^{\text{out}}$ at time step k. The set $\mathcal{N}_i^{\text{out}}$ is defined as

$$\mathcal{N}_i^{\text{out}} = \{(j,m) \mid l_{i,j}^m \text{ is an outgoing link for node } i\}.$$

  The value of $y_{i,j,o,d}^m(k)$ equals zero when $i = d$ (which implies that node i is actually the final destination node d of transport demand (o,d)).
- The value of $d_{i,o,d}^{\text{in}}(k)$ (TEUs/h) is the container flow corresponding to transport demand (o,d) and entering node i from the outside of the network at time step k. The value of $d_{i,o,d}^{\text{in}}(k)$ equals $d_{o,d}(k)$ when $i = o$, and otherwise it is zero.
- The value of $d_{i,o,d}^{\text{out}}(k)$ (TEUs/h) is the container flow corresponding to transport demand (o,d) and arriving at the final destination node i at time step k. The value of $d_{i,o,d}^{\text{out}}(k)$ equals $\sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k)$ when $i = d$ (here, we assume that containers coming from each transport demand will immediately leave the network once they arrive at their destination), and otherwise it is zero.
- The value of $t_{i,j}^m(k)T_{\text{s}}$ (h) is the transport time on link $l_{i,j}^m$ at time step k, and is given by

$$\begin{aligned} T_{i,j}^m(k) &= t_{i,j}^m(k)T_{\text{s}}, \\ t_{i,j}^m(k) &\in \mathbb{N}\backslash\{0\}, \\ t_{i,j}^m(k) &\leq t_{i,j}^{m,\max}, \end{aligned}$$

  where $t_{i,j}^{m,\max}$ is a positive integer that corresponds to $t_{i,j}^{m,\max}T_{\text{s}}$, the maximum transport time on link $l_{i,j}^m$.
- The value of $q_{i,j,o,d}^{m,\text{out}}(k)$ (TEUs/h) is the container flow corresponding to transport demand (o,d) and leaving link $l_{i,j}^m$ at time step k.
- The value of $q_{i,j,o,d}^{m,\text{in}}(k)$ (TEUs/h) is the container flow corresponding to transport demand (o,d) and entering link $l_{i,j}^m$ at time step k.
- The value of $x_{i,j,o,d}^m(k)$ (TEUs) is the number of containers corresponding to transport demand (o,d) and traveling in link $l_{i,j}^m$ at time step k.
- The values of h_i^{in} (TEUs/h) and h_i^{out} (TEUs/h) are the maximal container unloading and loading rates of the equipment in node i, respectively.
- The value of S_i (TEUs) is the storage capacity in node i.
- The value of $C_{i,j}^m$ (TEUs) is the transport or transfer capacity of link $l_{i,j}^m$.
- The value of $C_{i,j}^{m,\text{in}}$ (TEUs/h) is the maximal volume of container flows that can enter link $l_{i,j}^m$ at each time step.

The dynamics of the intermodal transport network comprise the dynamics of nodes given by (1), the dynamics of links given by (2)-(3), and the dynamics of the interactions among nodes and links in the network, given by (4)-(5). There are also some capacity constraints on nodes and links, given by (6)-(10). This model captures all possible flow assignments in intermodal freight transport, a particular/optimal flow assignment can be determined by solving an optimization problem subject to the corresponding user-supplied objective function.

The freight flow assignment determined should minimize the total transport time and the total delivery cost of transport demands. The objective function is defined as follows:

$$J = \alpha(J_1 + J_2) + J_3 + J_4 \tag{11}$$

with

$$J_1 = \sum_{(o,d) \in \mathcal{O}_{\text{od}}} w_{o,d} \left[\sum_{k=1}^{N-1} \left[\sum_{i \in \mathcal{V}} x_{i,o,d}(k)T_{\text{s}} \right. \right. \\ \left. \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(k)T_{\text{s}} \right] \right] \tag{12}$$

$$J_2 = \sum_{(o,d) \in \mathcal{O}_{\text{od}}} w_{o,d} \left[\sum_{i \in \mathcal{V}} x_{i,o,d}(N)r_{i,d} \right. \\ \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(N)r_{i,j}^{m,d} \right] \tag{13}$$

$$J_3 = \sum_{(o,d) \in \mathcal{O}_{\text{od}}} w_{o,d} \left[\sum_{k=1}^{N-1} \left[\sum_{i \in \mathcal{V}} x_{i,o,d}(k)T_{\text{s}}C_{i,\text{store}}(k) \right. \right. \\ \left. \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(k)T_{\text{s}}C_{i,j,\text{tran}}^m(k) \right] \right] \tag{14}$$

$$J_4 = \sum_{(o,d) \in \mathcal{O}_{\text{od}}} w_{o,d} \left[\sum_{i \in \mathcal{V}} x_{i,o,d}(N)c_{i,d} \right. \\ \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(N)c_{i,j}^{m,d} \right], \tag{15}$$

where
- The terms J_1, J_3 are the total transport time and the total delivery cost of transport demands $\mathcal{O}_{\text{od}}$ and the terms J_2, J_4 are the penalties on the unfinished transport demands at the end of the planning horizon.
- The value of $w_{o,d} \in (0,1]$ indicates the relative priority of the transport demand (o,d). The relation $\sum_{(o,d) \in \mathcal{O}_{\text{od}}} w_{o,d} = 1$ always holds.
- The value of $C_{i,\text{store}}(k)$ (€/TEU/h) is the cost associated with storing containers in the node i at time step k.
- The value of $C_{i,j,\text{tran}}^m(k)$ (€/TEU/h) is the transport or transfer cost, i.e., the cost that has to be paid for using a link to transport or transfer containers at time step k.
- The typical transport time and the typical delivery cost for containers being transported from node i to destination node d are $r_{i,d}$ (h/TEU) and $c_{i,d}$ (€/TEU), respectively[1].
- The typical transport time and the typical delivery cost for containers being transported from link $l_{i,j}^m$ to

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[1]The values of $r_{i,d}$ and $c_{i,d}$ can be obtained from statistical data.

destination node d is $r_{i,j}^{m,d}$ (h/TEU) and $c_{i,j}^{m,d}$ (€/TEU), respectively[2].

- The parameter α (€/h) is a conversion factor for converting the transport time to its equivalent monetary cost.
- The planning horizon is $N \cdot T_s$ (h) with $N \in \mathbb{N}\backslash\{0\}$.

Therefore, the optimal freight flow assignment problem can be formulated as the following linear optimization problem:

$$\min_{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}} J(\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}) \qquad (16)$$

$$\text{subject to} \qquad (1) - (10),$$

where

- $\tilde{\mathbf{x}}_1$ contains all $x_{i,o,d}(k)$, for $i \in \mathcal{V}, (o,d) \in \mathcal{O}_{od}, k = 1, \ldots, N$.
- $\tilde{\mathbf{x}}_2$ contains all $x_{i,j,o,d}^m(k)$, for $(i,j,m) \in \mathcal{E}, (o,d) \in \mathcal{O}_{od}, k = 1, \ldots, N$.
- $\tilde{\mathbf{y}}$ contains all $y_{i,j,o,d}^m(k)$, for $i \in \mathcal{V}, (j,m) \in \mathcal{N}_i^{\text{out}}, (o,d) \in \mathcal{O}_{od}, k = 1, \ldots, N$.
- $\tilde{\mathbf{u}}$ contains all $u_{j,i,o,d}^m(k)$, for $i \in \mathcal{V}, (j,m) \in \mathcal{N}_i^{\text{in}}, (o,d) \in \mathcal{O}_{od}, k = 1, \ldots, N$.

B. A model extension for the intermodal transport network

In the road network, a natural phenomenon is that traffic volumes on links influence the transport times on these links. In this paper, the fundamental diagram [14], [15] is adopted to model this phenomenon so as to obtain a model extension.

1) The fundamental diagram: The fundamental diagram of traffic flows on freeways represents the equilibrium of traffic flows using two out of three aggregated variables: the mean traffic flow q (veh/h), the mean traffic density ρ (veh/km/lane), and the mean speed v (km/h). The fundamental equation of traffic flow captures the relationship among these three variables [14], [15]:

$$q = \lambda \rho v. \qquad (17)$$

In (17), λ is the number of lanes on the freeway. The fundamental diagram has three variants: the speed-density fundamental diagram, the flow-density fundamental diagram, and the flow-speed fundamental diagram. These three types of fundamental diagram can be easily transformed from one type to another type based on the fundamental equation of traffic flow (17). The speed(v)-density(ρ) relation can be modeled as:

$$v(\rho) = v_{\text{free}} \exp\left[-\frac{1}{a}\left(\frac{\rho}{\rho_{\text{crit}}}\right)^a\right], \qquad (18)$$

where v_{free} is the free-flow speed, ρ_{crit} is the critical density (which corresponds to the maximum traffic flow), and a is a parameter in the model [14]. Some typical values of above three model parameters are: $v_{\text{free}} = 120$ km/h, $\rho_{\text{crit}} = 33.5$ veh/km/lane, and $a = 1.867$ [16], [17]. Figures 1 and 2 show the speed-density fundamental diagram and the flow-density fundamental diagram for a freeway with a single lane, respectively.

[2]The values of $r_{i,j}^{m,d}$ and $c_{i,j}^{m,d}$ can be obtained from statistical data.
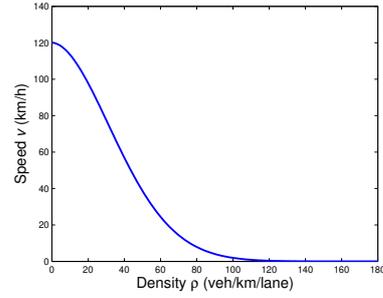

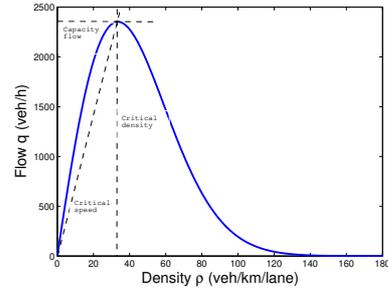Fig. 1. The speed-density fundamental diagram.

Fig. 2. The flow-density fundamental diagram.

2) Dynamics of links in the road network: From the intermodal freight transport point of view, traffic flows on freeways of the road network can be categorized into two parts: the freight truck flows and other traffic flows. The freight truck flows are caused by intermodal freight (container) transport. The combination of these two classes of traffic flows determines the traffic densities on freeways and subsequently the transport times needed to traverse them. In case of multi-class traffic flows, the concept of "equivalent vehicles" has been developed in [18] so as to take differences in the typical lengths of the vehicles for each class into account. In this paper, we adopt the same idea but formulate it in a simplified way. In particular, the total traffic density $\rho_{i,j}^{\text{truck}}(k)$ on link $l_{i,j}^{\text{truck}}$ at time step k can be derived as:

$$\rho_{i,j}^{\text{truck}}(k) = \theta \rho_{i,j}^{\text{truck,truck}}(k) + \rho_{i,j}^{\text{truck,oth}}(k) \quad (19)$$

$$\theta = \frac{L_{\text{truck}}}{L_{\text{oth}}} \qquad (20)$$

$$\rho_{i,j}^{\text{truck,truck}}(k) = \sum_{(o,d)\in\mathcal{O}_{od}} \frac{1}{L_{i,j}^{\text{truck}}\lambda_{i,j}^{\text{truck}}} x_{i,j,o,d}^{\text{truck}}(k) \quad (21)$$

where

- The actual traffic density corresponding to freight truck flow on link $l_{i,j}^{\text{truck}}$ at time step k is $\rho_{i,j}^{\text{truck,truck}}(k)$.
- The traffic density induced by the other traffic on link $l_{i,j}^{\text{truck}}$ at time step k is $\rho_{i,j}^{\text{truck,oth}}(k)$.
- The value of θ represents the relation between the typical length of trucks and that of other vehicles.
- The average length of freight trucks and other vehicles are L_{truck} (meters) and L_{oth} (meters), respectively.
- The length and the number of lanes of link $l_{i,j}^{\text{truck}}$ are $L_{i,j}^{\text{truck}}$ (km) and $\lambda_{i,j}^{\text{truck}}$, respectively.

We consider the freeway connections among intermodal

terminals in the intermodal transport network. The fundamental diagram can be deployed to model the evolution of traffic flows on links of the road network and then to derive the evolution of transport time for different classes of traffic flows on each link. Based on (18), the freight truck flow speed on link $l_{i,j}^{\text{truck}}$ at time step k, $v_{i,j}^{\text{truck,truck}}(k)$, and the corresponding transport time, $t_{i,j}^{\text{truck}}(k)$, can be derived as:

$$
v_{i,j}^{\text{truck,truck}}(k) = v_{i,j,\text{free}}^{\text{truck,truck}} \exp\left[-\frac{1}{a_{i,j}^{\text{truck,truck}}} \left(\frac{\rho_{i,j}^{\text{truck}}(k)}{\rho_{i,j,\text{crit}}^{\text{truck}}} \right)^{a_{i,j}^{\text{truck,truck}}} \right],
$$
(22)

$$
t_{i,j}^{\text{truck}}(k) = \mathbf{round}\left(\frac{L_{i,j}^{\text{truck}}}{v_{i,j}^{\text{truck,truck}}(k)} \cdot \frac{1}{T_{\text{s}}} \right),
$$
(23)

3) The optimal freight flow assignment problem: We consider the case in which pre-scheduled timetables are available for trains and barges on the railway network and the waterway network, respectively. However, the time needed for trucks to traverse links of the road network, depends on the traffic densities on these links. As has been presented in Section III-B, the travel times can be derived by using the fundamental diagram model for multi-class traffic flows on the road network. Therefore, the resulting optimal freight flow assignment problem can be formulated as:

$$
\min_{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\boldsymbol{\rho}}} J(\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\boldsymbol{\rho}}, \tilde{\boldsymbol{t}})
$$
(24)

subject to $(1) - (10), (19) - (23)$.

where $\tilde{\boldsymbol{\rho}}$ contains all $\rho_{i,j}^{\text{truck}}(k), \{i, j, \text{truck}\} \in \mathcal{E}, k = 1, \cdots, N$, $\tilde{\boldsymbol{t}}$ contains all $t_{i,j}^{\text{truck}}(k), \{i, j, \text{truck}\} \in \mathcal{E}, k = 1, \cdots, N$ and all other variables are the same as the optimization problem (16) in the generic intermodal transport network model in Section III-A. Obviously, in case of fixed transport times on links of the road network, the optimization problem (24) is reduced to the linear optimization problem (16).

IV. A SEQUENTIAL LINEAR PROGRAMMING APPROACH FOR DETERMINING OPTIMAL FREIGHT FLOW ASSIGNMENT

Because of the nonlinear equality constraints (22) and (23) induced by the fundamental diagram model, the optimal freight flow assignment problem (24) becomes a nonlinear and non-convex optimization problem, which is typically difficult to solve. Instead of directly solving this nonlinear and non-convex optimization problem, a sequential linear programming (SLP) approach is proposed to determine the freight flow assignment. The main idea is to iteratively conduct an optimization and update process such that the resulting value of the objective function J converges to a certain value. This optimization and update process consists of two parts: firstly, the optimal freight flow assignment (i.e., $\tilde{\mathbf{y}}$) and the value of the objective function J, are determined

for fixed link transport times (i.e., $t_{\text{fixed}}^{\text{train}}, t_{\text{fixed}}^{\text{barge}}, t^{\text{truck}}$) by solving the optimization problem (16); next, the transport times on links of the road network, t^{truck}, are updated using the resulting freight flow assignment (i.e., $\tilde{\mathbf{y}}$) obtained in the first optimization part on the basis of (22) and (23). The stopping threshold J_s and the maximum iteration number N_{max} are set as inputs of the iteration procedure. Note that in the first iteration transport times on links of the road network, t^{truck}, are initialized as their typical transport times. Because in the above iterative procedure a linear programming optimization problem (16) is solved multiple times in sequence, the proposed iteration procedure is called an SLP approach. This SLP approach is implemented using the iterative procedure given as follows:

Input : J_{s}, N_{max}, $\boldsymbol{t}_{\text{fixed}}^{\text{train}}$, $\boldsymbol{t}_{\text{fixed}}^{\text{barge}}$, $\boldsymbol{t}_{\text{typical}}^{\text{truck}}$, an intermodal transport network.
$J_{\text{c}} \leftarrow +\infty$
$n \leftarrow 1$
$\boldsymbol{t}^{\text{truck}} \leftarrow \boldsymbol{t}_{\text{typical}}^{\text{truck}}$
while $J_{\text{c}} \geq J_{\text{s}}$ and $n \leq N_{\text{max}}$ **do**
  $J, \tilde{\mathbf{y}} \leftarrow$ Solve optimization problem (16) for $\boldsymbol{t}_{\text{fixed}}^{\text{train}}, \boldsymbol{t}_{\text{fixed}}^{\text{barge}}, \boldsymbol{t}^{\text{truck}}$
  $\boldsymbol{t}^{\text{truck}} \leftarrow$ Actual transport times on links of the road network with $\boldsymbol{t}_{\text{fixed}}^{\text{train}}, \boldsymbol{t}_{\text{fixed}}^{\text{barge}}, \boldsymbol{t}^{\text{truck}}, \tilde{\mathbf{y}}$ based on equations (22) and (23)
  if $n = 1$ **then**
    $J_{\text{p}} \leftarrow J$
  else
    Compute $J_{\text{c}} \leftarrow \frac{|J - J_{\text{p}}|}{|J_{\text{p}}|}$
    $J_{\text{p}} \leftarrow J$
  end if
  $n \leftarrow n + 1$
end while

Linear programming problems can be solved very efficiently [19]. Therefore, the freight flow assignment in the extended intermodal transport network model can be determined quickly using the proposed SLP approach. But the global optimality of the freight flow assignment determined by the SLP approach is not guaranteed, since the optimal freight flow assignment problem (24) is basically a nonlinear and non-convex optimization problem.

V. SIMULATION STUDY

In this section, a simple simulation is conducted to determine the optimal freight flow assignment in intermodal freight transport with the SLP approach proposed in Section IV. First, we describe an intermodal freight transport problem. Next, we analyze the simulation results and discuss the efficiency of the proposed SLP approach.

A. An intermodal freight transport problem

We consider a simple intermodal transport network, consisting of three different types of transport networks. The set-up of the corresponding intermodal transport network model is shown in Figure 3. The network model consists of 10 nodes and 32 links. The transport/transfer times and transport
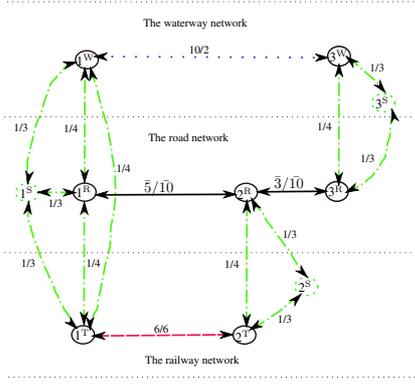
Fig. 3. The set-up of the simulation network model. Each double-headed arc in the figure represents two directed links with opposite directions.

TABLE I

THE TYPICAL TRANSPORT TIME $r_{i,d}$ (H)

	1^S	1^W	1^R	1^T	2^R	2^T	2^S	3^W	3^R	3^S
1^S	0	1	1	1	6	8	10	18	21	20
1^W	1	0	2	2	7	9	10	16	19	18
1^R	1	2	0	2	4	9	10	10	8	9
1^T	1	2	2	0	9	6	8	15	13	14
2^R	6	7	4	9	0	2	1	5	2	4
2^T	8	9	9	6	2	0	1	8	5	7
2^S	10	10	10	8	1	1	0	7	4	6
3^W	18	16	10	15	5	8	7	0	2	1
3^R	21	19	8	13	2	2	4	2	0	1
3^S	20	18	9	14	4	7	6	1	1	0

costs on links are shown as labels of each link in Figure 3. For example, label "10/2" for the transport link from node 1^W to node 3^W represents that it takes 10 hours to cross this link by barge and the transport cost is 2 €/TEUs/h. Note that for the road links $l^{\text{truck}}_{1^R,2^R}$ and $l^{\text{truck}}_{2^R,3^R}$ the link transport times are time-varying and load-dependent, and therefore the corresponding labels only show the typical transport times for these two road links. The typical transport times and the typical delivery costs between any pair of nodes of the network are given in Table I and Table II, respectively. The capacity parameters for nodes and links of the network are given in Tables III and IV.

We consider an intermodal freight transport process for 24 hours and choose the simulation time step, T_s, as one hour. Trains and barges are scheduled to depart at each hour from node 1^T and node 1^W, respectively. The traffic densities induced by other traffic flows on the road links $l^{\text{truck}}_{1^R,2^R}$ and $l^{\text{truck}}_{2^R,3^R}$ are given in Table V, and θ is taken as 2. A piecewise constant transport demand during the simulation period, as given in Table VI, enters the network from node 1^R to be transported to node 3^R. For the above intermodal freight transport setup, the initial state of the network is taken to be empty (e.g., $x_{i,o,d}(k) = 0$ and $x^m_{i,j,o,d}(k) = 0$ for $\forall(o,d) \in \mathcal{O}_{\text{od}}, (i,j,m) \in \mathcal{E}, k \leq 0$). As inputs of the SLP approach, we have the stopping threshold $J_s = 1.0 \cdot 10^{-4}$ and the maximum iteration number $N_{\max} = 5$.

B. Results and analysis

We use the *"interior point"* method of the *"linprog"* function in the Matlab Optimization Toolbox to optimize the intermodal freight transport process introduced in Section

TABLE II

THE TYPICAL TRANSPORT COST $c_{i,d}$ (€)

	1^S	1^W	1^R	1^T	2^R	2^T	2^S	3^W	3^R	3^S
1^S	0	3	3	3	44	40	45	24	29	28
1^W	3	0	4	4	45	41	46	20	25	24
1^R	3	4	0	4	40	41	45	25	37	41
1^T	3	4	4	0	41	36	40	25	30	29
2^R	44	45	40	41	0	4	3	25	20	24
2^T	40	41	41	36	4	0	3	30	25	29
2^S	45	46	45	40	3	3	0	29	24	28
3^W	24	20	25	25	25	30	29	0	4	3
3^R	29	25	37	30	20	25	24	4	0	3
3^S	28	24	41	29	24	29	28	3	3	0

TABLE III

NODE PARAMETERS

Nodes	h_i^{in} (TEUs/h)	h_i^{out} (TEUs/h)	S_i (TEUs)
$1^R, 2^R, 3^R, 1^T, 2^T$	1000	1000	100
$1^W, 2^W$	1000	1000	100
$1^S, 2^S, 3^S$	1000	1000	20000

TABLE IV

LINK PARAMETERS

Links	Road link (TEUs/h)	Rail link (TEUs/h)	Water link (TEUs/h)	Transfer link (TEUs/h)
$C_{i,j}^{m,\text{in}}$	50	65	70	100

TABLE V

DENSITIES OF OTHER TRAFFIC FLOWS ON ROAD LINKS

Period (hours)	$0-6$	$6-18$	$18-24$
$\rho^{\text{truck,oth}}_{1^R,2^R}$ (veh/km/lane)	2.5	28.0	5.0
$\rho^{\text{truck,oth}}_{2^R,3^R}$ (veh/km/lane)	2.5	28.0	5.0

TABLE VI

TRANSPORT DEMAND

Period (hours)	$0-7$	$7-24$
$d_{1^R,3^R}$ (TEUs/h)	135	0

TABLE VII

MODEL SPLIT RATES FOR DIFFERENT α

Modality	truck (%)	train (%)	barge (%)
$\alpha = 0.05$	0.00	0.00	100.0
$\alpha = 1.50$	15.87	0.00	84.13
$\alpha = 15.00$	39.15	5.29	55.56

V-A. The freight flow assignment is determined by using the proposed SLP approach. For different values of the conversion factor α, the resulting modal split rates in node 1^R, transport times on links $l^{\text{truck}}_{1^R,2^R}$ and $l^{\text{truck}}_{2^R,3^R}$, and the corresponding iteration processes of the SLP approach are given in Table VII, Figure 4 and Figure 5, respectively.

Figure 5 illustrates the freight flow assignment that is found in the first iteration when the conversion factor α has the value of 0.05. As shown in Table VII, the value of the conversion factor α influences the modality choices when determining the freight flow assignment. Actually, in case of $\alpha = 0.05$, only the (cheap) waterway network is chosen for serving the transport demand $d_{1^R,3^R}$. This means that the evolution of the transport times on link $l^{\text{truck}}_{1^R,2^R}$ and $l^{\text{truck}}_{2^R,3^R}$ has no influence on the optimal freight flow assignment. Therefore, the nonlinear equality constraints (22) and (23)
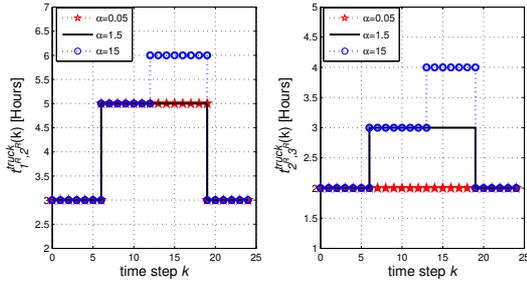
Fig. 4. $t_{1\text{R},2\text{R}}^{\text{truck}}(k)$ and $t_{2\text{R},3\text{R}}^{\text{truck}}(k)$ for different α.
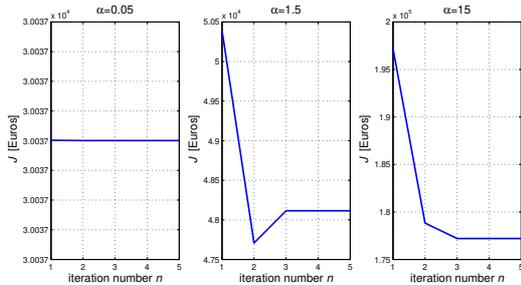

Fig. 5. The iteration process of the SLP approach for different α.

are actually redundant and the SLP approach can determine the freight flow assignment in the first iteration.

As the value of the conversion factor α increases, correspondingly the equivalent monetary costs of J_1 and J_2 contribute to a larger part of the optimization criterion (11). The (fast) road and railway networks are now also selected for delivering the transport demand $d_{1\text{R},3\text{R}}$. This explains the simulation result that in Table VII the market shares of trucks and trains increase for $\alpha = 1.50$ and $\alpha = 15.00$. For these two conversion factors, the time-varying and load-dependent transport times (see Figure 4) on $l_{1\text{R},2\text{R}}^{\text{truck}}$ and $l_{2\text{R},3\text{R}}^{\text{truck}}$ have an influence on the freight flow assignment. Figure 5 shows that the freight flow assignments are determined through the proposed SLP approach in the third iteration for $\alpha = 1.50$ and $\alpha = 15.00$. In the simulation, we have noticed that actually the proposed SLP approach almost always converges within 3 iterations. Note that the freight flow assignment determined by the proposed SLP approach will also be influenced by other factors, e.g., the typical transport times on road links. We leave the investigation of the degree of optimality of the freight flow assignment resulting from the proposed SLP approach as part of our future work.

VI. Conclusions and future work

An extended intermodal transport network model has been developed to model the time-varying and load-dependent transport time on links of the road network. A sequential linear programming approach has been proposed for solving the nonlinear and non-convex optimization model. In our future research, we will consider the load-dependent transport times on the waterway and rail networks and conduct more extensive case studies. We will also investigate a receding horizon approach for assigning intermodal freight flows.

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