

Technical report 09-022

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To cite this work, please refer to the published version:

G. Bajracharya, T. Koltunowicz, R. R. Negenborn, Z. Papp, D. Djairam, B. De Schutter, and J. J. Smit, "Optimization of maintenance for power system equipment using a predictive health model," *Proceedings of the 2009 IEEE Bucharest Power Tech Conference*, Bucharest, Romania, June–July 2009. Paper 563. doi:[10.1109/PTC.2009.5281928](https://doi.org/10.1109/PTC.2009.5281928)

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* This report can also be downloaded via <https://dpub.eu/09-022>

Optimization of Maintenance for Power System Equipment Using a Predictive Health Model

G. Bajracharya, T. Koltunowicz, R. R. Negenborn, Z. Papp, D. Djairam, B. De Schutter, J. J. Smit

Abstract—In this paper, a model-predictive control based framework is proposed for modeling and optimization of the health state of power system equipment. In the framework, a predictive health model is proposed that predicts the health state of the equipment based on its usage and maintenance actions. Based on the health state, the failure rate of the equipment can be estimated. We propose to use this predictive health model to predict the effects of different maintenance actions. The effects of maintenance actions over a future time window are evaluated by a cost function. The maintenance actions are optimized using this cost function. The proposed framework is applied in the optimization of the loading of transformers based on the thermal degradation of the paper insulation.

Index Terms—Power System Maintenance, Maintenance Optimization, Predictive Health Management, Model-Predictive Optimization, Power Transformer.

I. INTRODUCTION

In the power grid, a significant portion of the electrical infrastructures will reach the end of their operational age within the coming few decades. On the one hand, the impending replacement wave of these infrastructures will require extensive investments in the near future. On the other hand, the aging infrastructures are degrading the reliability of the system. So, there is a greater need for reducing the threat of the aging related failures and at the same time deferring the new investments by extending the life of the aging infrastructures. The extension of the life of the aging infrastructures should be done while keeping the reliability above an acceptable threshold. The performance, risk, and expenditures in the electrical infrastructure should thus be optimally managed to achieve quality of service in the most cost effective manner [1].

Maintenance is important for maintaining reliability of the equipment and extending the life of the equipment. Maintenance strategies implemented in electrical equipment can be categorized into three classes [1, 2]:

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- 1) Corrective Maintenance: Maintenance is performed only after breakdown of equipment.
- 2) Time-Based Maintenance: Maintenance is performed at predefined/fixed time steps.
- 3) Condition-Based Maintenance: Maintenance is based on the condition of the equipment.

Condition-based maintenance is becoming popular in electrical infrastructures over the traditional time-based maintenance [2]. Condition-based maintenance reduces the cost by performing maintenance only when it is needed. Currently, condition-based maintenance strategies are often based on heuristics. Knowledge rules and standards are used for the condition assessment and the maintenance is based on this condition assessment [3]. The rules and standards are developed based on expert knowledge and/or the analysis of the performance history of a set of identical equipment.

A model of the effects of maintenance actions on the health state of the equipment and its performance (i.e., reliability) is required to evaluate maintenance strategies [4]. Such a model emulates the evolution of the stresses in the equipment based on the physical principles of the aging mechanisms of the equipment. This model can be used to predict the effects of different planned maintenance strategies. These effects can be evaluated by associating cost functions to the health state and the performance of the equipment. An optimal maintenance strategy can then be devised by determining the effectiveness of different maintenance strategies using simulation and selecting the best strategy based on the cost associated with this strategy.

A framework of a predictive health model and the optimization of maintenance action are proposed in this paper. The framework is based on model predictive control in which the health model is used to predict the effect of the maintenance (control) actions. The framework is implemented in a case study of optimization of the loading of transformers.

The outline of this paper is as follows. In Section II, the proposed framework for the model-based optimization is presented. Section III gives a description of the aging of the paper insulation in transformers. An application of the framework for the case study and results are presented in Section IV. Conclusions and future work are included in Section V.

II. PROPOSED FRAMEWORK FOR MODEL-BASED OPTIMIZATION

The framework for model-based optimization that we propose here uses a predictive health model. Using the predictive

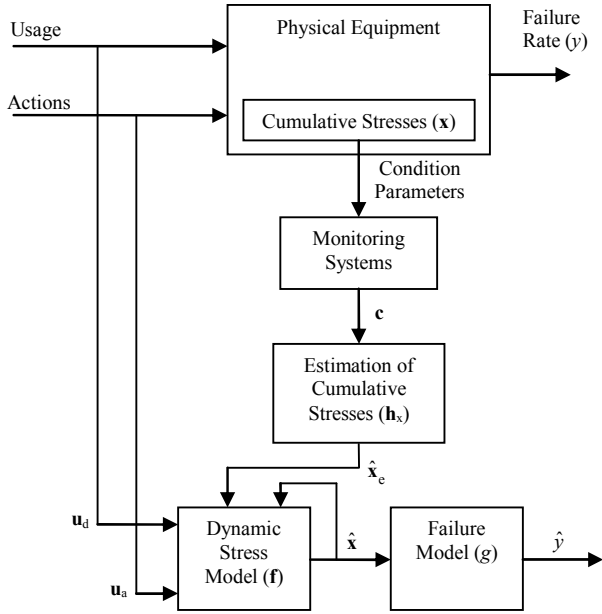


Fig. 1. Predictive health model

health model, the future health state of equipment used in the electricity grid can be predicted given possible actions and usage of the equipment. The framework also defines the cost function for the optimization.

A. Predictive Health Model

The predictive health model consists of the dynamic stress model, the failure model, and the estimation of cumulative stresses as illustrated in Fig. 1. As equipment ages, various stresses, such as electrical, thermal, mechanical, and environmental stresses, weaken the strength of the equipment. The cumulative stresses of the equipment are affected by the usage pattern (e.g., loading) and maintenance actions (e.g. replacement of parts) performed on the equipment. The health state of the equipment is represented by the cumulative stresses. Their dynamics can be described by a dynamic stress model which is implemented as a discrete-time state-space model. This model can predict future cumulative stresses $\hat{\mathbf{x}}(k+1)$ based on planned usage of the equipment $\mathbf{u}_d(k)$, planned maintenance actions $\mathbf{u}_a(k)$, and current cumulative stresses $\hat{\mathbf{x}}(k)$, where k is the current discrete time step. The dynamic stress model represents the aging of the equipment in which the cumulative stresses $\hat{\mathbf{x}}$ represent the health state of the equipment. The dynamic stress model is described as follows:

$$\hat{\mathbf{x}}(k+1) = \mathbf{f}(\hat{\mathbf{x}}(k), \mathbf{u}(k)), \quad (1)$$

where $\mathbf{u}(k) = [\mathbf{u}_a(k) \ \mathbf{u}_d(k)]^T$.

As the cumulative stresses increase over time, the probability of failure of the equipment also increases. The relationship between the cumulative stresses and the failure rate of the equipment is described in a failure model. The failure model uses the predicted cumulative stresses $\hat{\mathbf{x}}(k)$ to predict the failure rate $\hat{y}(k)$ of the equipment. The failure model directly

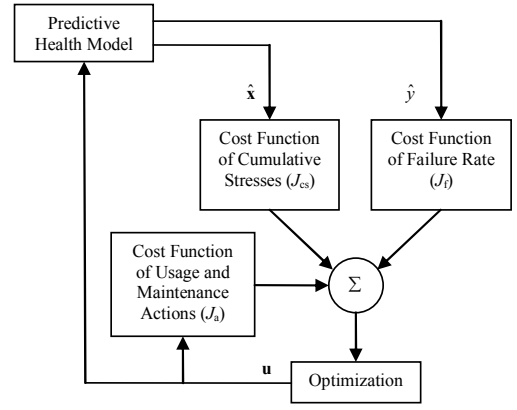


Fig. 2. Optimization of maintenance

maps the cumulative stresses to the failure rate as follows:

$$\hat{y}(k) = g(\hat{\mathbf{x}}(k)) \quad (2)$$

The cumulative stresses are indicated by condition parameters of the equipment, such as the partial discharge, temperature measurements, etc. Different online and offline monitoring systems can detect the condition parameters. In practice, only few condition parameters (e.g., electrical and thermal stresses) are measured by the monitoring systems. Estimates of the monitored cumulative stresses $\hat{\mathbf{x}}_e(k)$ can be made based on measurements $\mathbf{c}(k)$ of the monitoring systems as follows:

$$\hat{\mathbf{x}}_e(k) = \mathbf{h}_x(\mathbf{c}(k)) \quad (3)$$

The estimated cumulative stresses $\hat{\mathbf{x}}_e$ can be used in the dynamic stress model to update the corresponding cumulative stresses. The remaining unmonitored cumulative stresses are predicted by the dynamic stress model.

The framework of the predictive health model can be used to predict the health state and the failure rate of the equipment by considering the usage and the maintenance actions. The measurements of the monitoring systems can be used to update the cumulative stresses of the equipment.

B. Optimization of Maintenance

Typically, maintenance improves the health state of the equipment, which, in turn, reduces its failure rate. The optimal maintenance action balances the economical cost of the maintenance, the improvement of the health state, and the reduction in the failure rate of the equipment.

The process of model-based optimization is illustrated in Fig. 2. The total cost of the maintenance actions consists of three cost functions. The cost function of the planned usage and the maintenance actions J_a incorporates the economical cost of the maintenance. The cost function of the failure rate J_f takes into account the cost associated with the failure of the equipment. The cost function of the cumulative stresses J_{cs} incorporates the cost of deterioration of the equipment. The costs can be represented in monetary terms or can be normalized to the cost of the equipment. The summation of

these three different costs gives the total cost of a particular maintenance action in a particular state. The optimization of the maintenance actions is considered over a given time horizon in the future (N) so that the future maintenance actions can be optimized. The total cost over the time horizon is considered for the optimization. The optimization problem is formulated as follows:

$$\begin{aligned} \min_{\mathbf{u}_a(k), \dots, \mathbf{u}_a(k+N-1)} & \left[\sum_{l=0}^{N-1} J_a(\mathbf{u}(k+l)) \right] \\ & + \left[\sum_{l=0}^{N-1} J_f(\hat{\mathbf{y}}(k+l)) \right] \\ & + J_{cs}(\hat{\mathbf{x}}(k), \hat{\mathbf{x}}(k+N)), \end{aligned} \quad (4)$$

subject to

$$\begin{aligned} \hat{\mathbf{x}}(k+l+1) &= \mathbf{f}(\hat{\mathbf{x}}(k+l), \mathbf{u}(k+l)) \\ \hat{\mathbf{y}}(k+l) &= \mathbf{g}(\hat{\mathbf{x}}(k+l)), \quad \text{for } l=0, \dots, N-1. \end{aligned}$$

The predictive health model is used to predict the cumulative stresses and the failure rates for the planned usage pattern and different future maintenance actions. The total cost is calculated for different future usage and maintenance actions over the time horizon. Optimal maintenance actions minimizing the total cost over the time horizon are searched.

III. DESCRIPTION OF THE TRANSFORMER MODEL

The proposed framework has been implemented on a case study of transformer insulation systems. Such a transformer insulation system consists of cellulose paper impregnated with mineral oil of the transformer. In this particular case, the use of the framework is illustrated by considering the health state of paper insulation only. A model of the degradation of the paper insulation is used to determine the optimal loading of the transformer. The loading is taken as a planned usage of the equipment.

A. Aging Model of the Insulation System of a Transformer

A transformer consists of various sub-components, such as windings, cellulose paper insulation, a core, tap changers, etc. The health of a transformer depends on the health state of its sub-components. One of the important sub-components is the cellulose paper insulation. Degradation of the cellulose paper insulation, due to thermal stress, oxidation, and hydrolytic processes reduces its dielectric and mechanical strength. This cellulose degradation determines the ultimate life of the insulation system [5].

The degradation process depends mainly on the temperature. Different models have been proposed to investigate the effects of the temperature on the aging of cellulose paper. The International Electrotechnical Commission Loading Guide [6] uses the hottest spot winding temperature to predict the life of the insulation system. Emsley et al. [5] have proposed kinetics of degradation of the cellulose paper based on its degree of polymerization. The degree of polymerization is the average chain length of the polymer in the cellulose. A decrease in the degree of polymerization signifies degradation of the paper.

According to the degradation model from Emsley et al. [5], the degree of polymerization of cellulose paper can be estimated by the following equation:

$$\frac{1}{DP_t} - \frac{1}{DP_0} = A \exp\left(-\frac{E}{R(T+273)}\right) \times 24 \times 365 \times t, \quad (5)$$

where DP_t and DP_0 are the value of the degree of polymerization at time t and 0, respectively, A is a pre-exponential constant, E is the activation energy, R ($= 8.314 \text{ kJ mole}^{-1} \text{ K}^{-1}$) is the gas constant, T is the temperature of the cellulose paper in Celsius, and t is the elapsed time in years.

For dry Kraft paper in oil, which is commonly used in transformers, the activation energy E can be taken as 111 kJ mole^{-1} and the pre-exponential constant A can be taken as 1.07×10^8 [5].

The degree of polymerization of new paper insulation may vary from 1300 to 900. The paper is considered to be at the end of its life if its degree of polymerization reaches between 150 and 250 [5]. For the model used in this paper, an initial value of 1000 and a final value of 200 are used [1], [7].

The degradation model (5) will be used to estimate the effect of the temperature on the condition of the paper.

B. Relationship between Temperature and Loading

The temperature of the insulation depends upon the ambient temperature and the heat generated due to the power losses in the transformer. The power losses consist of iron losses and copper losses. The iron losses are almost constant for the normal operation whereas the copper losses increase with the increase in the current (loading) of the transformer. The loading of the transformer can vary on a daily, weekly, and seasonal basis.

Different models have been proposed to estimate the hottest spot winding temperature in transformers. The temperature can be estimated by the thermal model based on heat transfer [8], [9]. Various measurements, such as the top-oil temperature and the bottom-oil temperature, are also used for the estimation [10]. The steady state hottest spot winding temperature T can be calculated as follows [8]:

$$T = \theta_A + \Delta\theta_{TO,R} \left(\frac{K_U^2 R + 1}{R + 1} \right)^n + \Delta\theta_{H,R} (K_U^2)^m, \quad (6)$$

where θ_A is the ambient temperature, K_U is per unit load, $\Delta\theta_{TO,R}$ and $\Delta\theta_{H,R}$ are the rated top oil rise over the ambient temperature and the rated hottest spot winding temperature rise over the top oil temperature, respectively, R is the ratio of the load loss at the rated load to the no-load loss, n and m are empirical values which depend on the type of cooling of the transformer.

For a 187 MVA transformer described in Annex C of [8], $\Delta\theta_{TO,R}$ and $\Delta\theta_{H,R}$ are considered as 36°C and 28.6°C respectively. R is calculated as 4.87 and θ_A is assumed as 30°C . The values of both n and m are considered as 1 for the directed forced-oil cooling of the transformer. The steady state hottest spot winding temperature with respect to loading is calculated by (6), which is shown in Fig. 3.

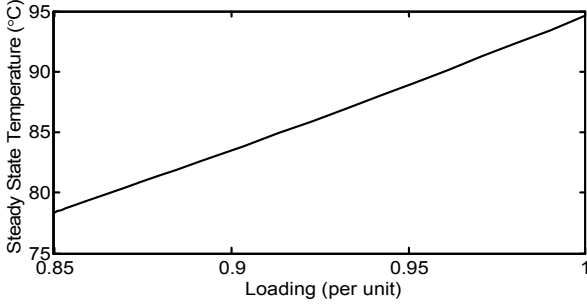


Fig. 3. Relation of the loading to the steady state hottest spot winding temperature

IV. APPLICATION OF THE TRANSFORMER MODEL TO THE MODEL-BASED OPTIMIZATION FRAMEWORK

A. A Predictive Health Model of Cellulose Paper Insulation

In the predictive health model of the cellulose paper insulation, the degree of polymerization \hat{x}_{DP} is taken as the cumulative stress. The temperature of the transformer u_T depends upon the loading and thus can be considered as a planned usage of the equipment. This way, the model predicts the effect of the loading of the transformer on the degree of polymerization of the paper insulation. The kinetics of degradation of the degree of polymerization described in (5) are discretized to obtain the dynamic stress model, as described in (1). The dynamic stress model, discretized for a time step of 1 year, is then as follows:

$$\frac{1}{\hat{x}_{DP}(k+1)} = \frac{1}{\hat{x}_{DP}(k)} + A \exp\left(-\frac{E}{R(u_T(k) + 273)}\right) \times 24 \times 365 \times h, \quad (7)$$

where $\hat{x}_{DP}(k)$ and $u_T(k)$ represent the degree of polymerization and the temperature of the insulation system in the k^{th} year, respectively, and h is the time step (1 year) of the discrete-time model.

The failure model for transformers due to the insulation degradation is not considered in this paper. The contribution to the failure rate due to the insulation degradation is relatively low compared to the failure rate due to other components, such as tap changers [11]. However, the degradation of the insulation system leads to the end of the operating life of the transformer and thus has a major impact in terms of the investment of the equipment [12]. Thus, only the dynamic stress model is considered for the optimization.

B. Cost functions

The cost function of the cumulative stresses (i.e., the degree of polymerization) J_{cs} and the cost function of the usage (i.e., the temperature) J_a have been developed for the optimization. J_{cs} accounts for the cost of aging of the equipment. A linear depreciation of the cost of the equipment with respect to the operating time is commonly used. The cost of the degree of

polymerization is considered as 1 (normalized) when the degree of polymerization is reduced to the minimum acceptable value (i.e., $DP_{\text{final}} = 200$). The initial degree of polymerization of the new paper insulation DP_{initial} is considered to be 1000. The cost function for the degree of polymerization is as follows:

$$J_{cs}(\hat{\mathbf{x}}(k), \hat{\mathbf{x}}(k+N)) = \alpha \left(\frac{1}{\hat{x}_{DP}(k+N)} - \frac{1}{\hat{x}_{DP}(k)} \right), \quad (8)$$

where $\alpha = \frac{1}{1/DP_{\text{final}} - 1/DP_{\text{initial}}} = 250$.

The cost function of the temperature J_a describes the cost of the loading of the transformer (see Fig. 3). A linear function is assumed for the cost function. A linear coefficient β represents the cost due to the decrement of the temperature (i.e., the benefit due to the increment of the temperature). A reference temperature T_{ref} is considered and the cost is calculated with respect to this reference temperature. Thus the cost is a relative cost with respect to the reference temperature. The cost function is given as:

$$J_a(u_T(k)) = -\beta(u_T(k) - T_{\text{ref}}). \quad (9)$$

C. Optimization

The predictive health optimization problem is obtained by substituting J_{cs} and J_a from (8) and (9), respectively, in (4). J_f is not included as the failure model is not considered. The constraints of the optimization are obtained from the predictive health model, described by (7). The resulting optimization problem is:

$$\min_{u_T(k), \dots, u_T(k+N-1)} \left[\sum_{l=0}^{N-1} -\beta(u_T(k+l) - T_{\text{ref}}) \right] + \alpha \left(\frac{1}{\hat{x}_{DP}(k+N)} - \frac{1}{\hat{x}_{DP}(k)} \right), \quad (10)$$

subject to

$$\frac{1}{\hat{x}_{DP}(k+l+1)} = \frac{1}{\hat{x}_{DP}(k+l)} + A \exp\left(-\frac{E}{R(u_T(k+l) + 273)}\right) \times 24 \times 365 \times h, \quad \text{for } l = 0, \dots, N-1.$$

The optimization is performed over a prediction horizon of N years. In the optimization, the replacement of the transformer in the case of its end of life (degree of polymerization less than 200) is not considered. Thus the prediction horizon, N , has to be less than the minimum expected life within the given range of parameters. For instance, in the case of the maximum temperature of 95°C , the minimum expected life is approximately 25 years (see Fig. 4). Thus N is chosen as 20 years. The reference temperature T_{ref} is taken as the mid-range value (87.5°C) of the investigated temperature range of 80°C to 95°C . The selection of the reference temperature T_{ref} does not affect the optimal solution. The solution to the optimization problem gives the optimal temperature that yields the minimum total cost.

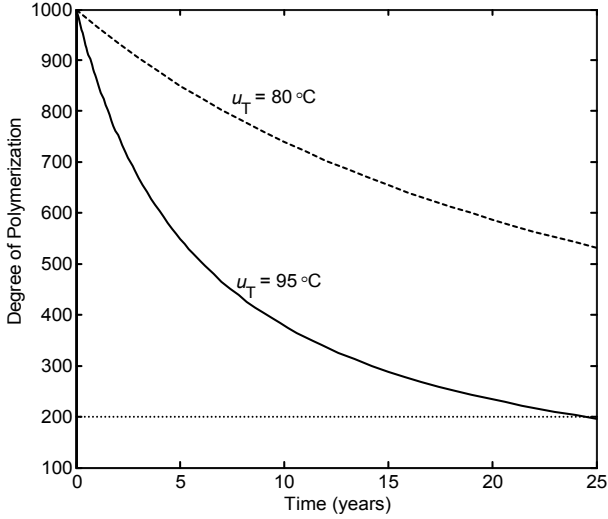


Fig. 4. Degree of polymerization for different temperatures

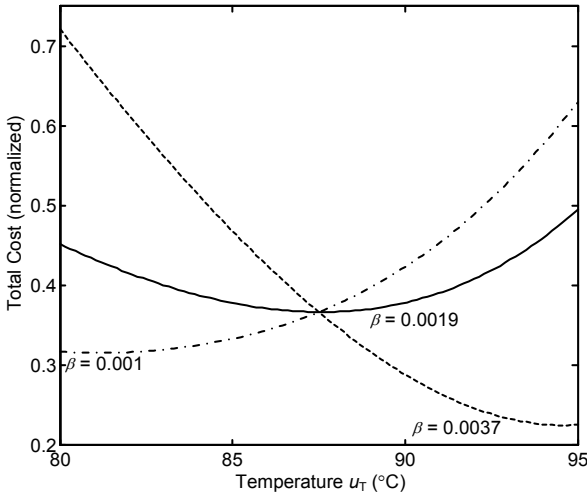


Fig. 5. Total cost for different values of cost coefficient β

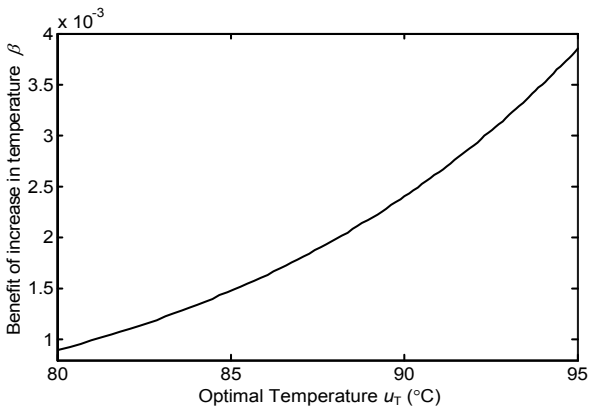


Fig. 6. Optimal temperature for different cost coefficient β .

The optimization problem (10) consists of a non-linear cost function and non-linear constraints. The optimization therefore is solved by a non-linear solver, SNOPT [13]. The solver is used through the Tomlab v6.1 [14] interface in Matlab v7.5.

D. Results

For a transformer, as the temperature increases, the insulation is stressed further, resulting in faster aging. An increase in temperature implies an increase in the loading of the transformer. Thus as the loading of the transformer is increased, the rate of aging is also increased and vice versa. The optimal solution is found when the cost of aging and the benefit of the increase in the loading are matched. The optimal solution depends upon how important or unimportant the increase in the loading is compared to the loss of life of the insulation system. The criticality of the loading depends upon various factors including the importance of the transformer, the criticality of the transformer location, and the loading profile of the transformer.

The benefit of the increase in the loading (or the increase in the temperature) is quantified by the benefit due to the increment of the temperature β in (10). Fig. 5 illustrates the total cost with respect to the temperature u_T for three different values of β . As observed in the figure, the optimal temperature varies as β is changed.

The optimal temperature for different values of β is plotted in Fig. 6. A constant temperature over the prediction horizon is considered since the purpose is to obtain the optimal temperature. As illustrated in the figure, if the benefit due to the increment of the temperature is lower, a lower temperature is optimal and vice versa.

The results of the optimization can be used for the loading of the transformer. As the temperature is related to the loading of the transformer (see Fig. 3), the optimal loading varies as the value of β changes. The value of β can be chosen according to the importance of the transformer and the priorities of the utility. The recommendation of the future loading of the transformer, which is a part of the maintenance planning, can be made based the optimization result.

V. CONCLUSIONS AND FUTURE WORK

A framework for predictive health modeling of power system equipment has been proposed and a model-based optimization has been implemented for the optimization of maintenance. The framework is applicable to most of the power system equipments. A case study of optimization of the loading of transformers based on the framework has been proposed in this paper.

The presented framework combines a prediction model based on the aging mechanisms as well as an estimation of the health state based on online and offline monitoring systems. The business values, such as the performance of the equipment and the investment cost in maintenance, have been translated into the cost functions of the optimization.

The framework will be extended to include the hottest spot winding temperature estimation. For the estimation, the time

step of the model will be decreased to match the dynamics of the temperature. The framework will also be extended for the system-wide optimization of maintenance over a network. Distributed problem solving and optimization schemes will be applied for the system-wide optimization. Furthermore, the model presented can be extended to include a multi-component model that considers aging due to other components, such as the oil condition.

VI. ACKNOWLEDGMENT

This research is supported by the SenterNovem Sinergie project EOSLT04034, the BSIK project "Next Generation Infrastructures (NGI)", the Delft Research Center Next Generation Infrastructures, the European STREP project "Hierarchical and distributed model predictive control (HD-MPC)", and the project "Multi-Agent Control of Large-Scale Hybrid Systems" (DWV.6188) of the Dutch Technology Foundation STW.

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