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Adaptive Model Predictive Control using max-plus-linear input-output models

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Abstract

Model predictive control (MPC) is a popular controller design technique in the process industry. Conventional MPC uses linear or nonlinear discrete-time models. Recently, we have extended MPC to a class of discrete event systems that can be described by a model that is "linear" in the max-plus algebra. In our previous work we have considered MPC for the time-invariant case. In this paper we consider an adaptive scheme for the time-varying case, based on parameter estimation of input-output models. In a simulation example we show that the combined parameter-estimation/MPC algorithm gives a good closed-loop behaviour.

1 Introduction

Clarke *et al.* [3] and Mosca [11] demonstrate how predictive control can provide adaptive controllers. The predictive technique is seen as a tool to go beyond the conventional single-step-ahead adaptive control strategies. Model predictive control (MPC) [8] is a proven technology for the control of multivariable systems in the presence of input and output constraints and is capable of tracking prescheduled reference signals. At each time instant the process model is updated, based on measured input and output data. On the basis of this model, predictions of the process signals over a specified horizon are made to optimize the future control signal by minimizing a performance index. The resulting controller is called an adaptive model predictive controller.

Usually adaptive MPC uses linear or nonlinear discrete-time models. However, the attractive features mentioned above have led us to extend the adaptive MPC scheme to discrete event systems. Typical examples of discrete event systems (DES) are flexible manufacturing systems, telecommunication networks, parallel processing systems, traffic control systems, and logistic systems. There exist many different modeling and analysis frameworks for DES such as Petri nets, finite state machines, automata, languages, process algebra, computer models, etc. [2, 7]. In this paper we consider the class of DES with synchronization but no concurrency. Such DES can be described by models that are "linear" in the max-plus algebra [1, 4], and therefore they are called max-plus-linear (MPL) DES. In [5] we have derived an MPC controller for this framework and we have also shown that under quite general conditions the resulting MPC optimization problem is a convex optimization problem. This paper describes an adaptive MPC methodology for slowly time-varying MPL systems using an input-output model. An input-output setting is used because in many applications only input and output measurements are available. In this paper we consider the noise-free case.

2 Max-plus-linear input-output systems

In this section we define the class of MPL input-output systems. For this purpose we will first give the basic definition of the max-plus algebra and min-plus algebra, and we present some results for max-plus polynomials.

Max-plus algebra

Define $\varepsilon = -\infty$ and $\mathbb{R}_\varepsilon = \mathbb{R} \cup \{\varepsilon\}$. The max-plus-algebraic addition ($\oplus$) and multiplication ($\otimes$) are defined as follows [1, 4]:

$$x \oplus y = \max(x, y) \qquad x \otimes y = x + y$$

for numbers $x, y \in \mathbb{R}_\varepsilon$, and

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij})$$
$$[A \otimes C]_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \max_{k=1,\ldots,n}(a_{ik} + c_{kj})$$

for matrices $A, B \in \mathbb{R}_\varepsilon^{m\times n}$ and $C \in \mathbb{R}_\varepsilon^{n\times p}$.

Min-plus algebra

Define $\top = \infty$ and $\bar{\mathbb{R}} = \mathbb{R}_\varepsilon \cup \{\top\} = \mathbb{R} \cup \{\varepsilon, \top\}$. The min-plus-algebraic addition ($\oplus'$) and multiplication ($\otimes'$) are defined as follows [1, 4]:

$$x \oplus' y = \min(x, y) \qquad x \otimes' y = x + y$$

for numbers $x, y \in \bar{\mathbb{R}}$. By definition $\varepsilon \otimes \top = \top \otimes \varepsilon = \varepsilon$ and $\varepsilon \otimes' \top = \top \otimes' \varepsilon = \top$. For matrices $A, B \in \bar{\mathbb{R}}^{m\times n}$ and $C \in \bar{\mathbb{R}}^{n\times p}$ we have

$$[A \oplus' B]_{ij} = a_{ij} \oplus' b_{ij} = \min(a_{ij}, b_{ij})$$
$$[A \otimes' C]_{ij} = {\bigoplus_{k=1}^{n}}' a_{ik} \otimes' c_{kj} = \min_{k=1,\ldots,n}(a_{ik} + c_{kj})$$

Max-plus polynomials

This section is based on Bacelli *et al.* [1]. Define the delay operator γ as $\gamma\, z(k) = z(k-1)$. Now we can define the max-plus polynomial

$$P(\gamma) = p_0 \otimes \gamma^0 \oplus p_1 \otimes \gamma^1 \oplus \ldots \oplus p_n \otimes \gamma^n$$

where n is the order of the polynomial. We obtain

$$\begin{aligned} &P(\gamma)\, z(k) \\ &= \Big(p_0 \otimes \gamma^0 \oplus p_1 \otimes \gamma^1 \oplus \ldots \oplus p_n \otimes \gamma^n\Big) z(k) \\ &= p_0 \otimes z(k) \oplus p_1 \otimes z(k-1) \oplus \ldots \oplus p_n \otimes z(k-n) \end{aligned}$$

Let P, Q be nth order max-plus polynomials and let R be a mth order max-plus polynomial. The max-plus product and max-plus sum for polynomials are defined as follows:

$$P(\gamma) \oplus Q(\gamma) \;=\; \bigoplus_{i=0}^{n} (p_i \oplus rq_i) \otimes \gamma^i$$
$$P(\gamma) \otimes R(\gamma) \;=\; \bigoplus_{i=0}^{n} \bigoplus_{j=0}^{m} (p_i \oplus r_j) \otimes \gamma^{i+j}$$

Max-plus-linear Input-Output systems

In [5] we have used a state-space setting to study DES in which there is synchronization but no concurrency. In this paper we will consider these systems in an input-output setting. Our motivation behind this is that in practice only input-output signals are available, and the input-output form gives a compact description of the system. Consider systems that can be described by the input-output relation

$$\begin{aligned} y(k) \;=\; & a_1 \otimes y(k-1) \oplus \ldots \oplus a_n \otimes y(k-n) \oplus \\ & b_0 \otimes u(k) \oplus \ldots \oplus b_m \otimes u(k-m) \end{aligned}$$

This can be rewritten in polynomial form as

$$y(k) = A(\gamma) y(k) \oplus B(\gamma) u(k) \tag{1}$$

where $A(\gamma)$ and $B(\gamma)$ are polynomial operators

$$\begin{aligned} A(\gamma) \;&=\; a_1 \otimes \gamma^1 \oplus a_2 \otimes \gamma^2 \oplus \ldots \oplus a_n \otimes \gamma^n \\ B(\gamma) \;&=\; b_0 \otimes \gamma^0 \oplus b_1 \otimes \gamma^1 \oplus \ldots \oplus b_m \otimes \gamma^m \end{aligned} \tag{2}$$

DES that can be described by this model will be called max-plus-linear input-output (MPLIO) systems. The index k is called the event counter. The input $u(k)$ contains the time instants at which the input events occur for the kth time, and the output $y(k)$ contains the time instants at which the output events occur for the kth time[1]. The entries of system polynomials $A(\gamma)$ and $B(\gamma)$ are varying in time due to slow changes in the model.

3 Identification of MPLIO systems

Consider the SISO[2] MPLIO model, described by the input-output relation (1) and (2). We assume that the 'real' system is in the model set, and we denote the estimates of the input-output polynomials from (1) by $\hat{A}(\gamma)$ and $\hat{B}(\gamma)$. The prediction error $\xi(k)$ after the measurements of the kth event is then defined as

$$\xi(k) \;=\; y(k) - \Big(\hat{A}(\gamma) y(k) \oplus \hat{B}(\gamma) u(k)\Big)$$

[1]More specifically, for a manufacturing system, $u(k)$ contains the time instants at which the kth batch of raw material is fed to the system, and $y(k)$ the time instants at which the kth batch of finished product leaves the system.

[2]For sake of simplicity SISO systems are considered in this paper. However, all results can easily be extended to the MIMO case.

$$= y(k) - \Big(\underbrace{\begin{bmatrix} \hat{a}_1 & \cdots & \hat{a}_n & \hat{b}_0 & \cdots & \hat{b}_m \end{bmatrix}}_{\hat{\theta}} \otimes \underbrace{\begin{bmatrix} y(k-1) \\ \vdots \\ y(k-n) \\ u(k) \\ \vdots \\ u(k-m) \end{bmatrix}}_{p(k)} \Big)$$

$$= y(k) - \hat{\theta} \otimes p(k)$$

The elements of the vector $\hat{\theta}$ are estimates of the system parameters. Considering k consecutive events, i.e. the measurement data of k process cycles, one obtains the prediction error matrix

$$\underbrace{\begin{bmatrix} \xi(k) & \cdots & \xi(1) \end{bmatrix}}_{\Xi(k,1)} = \underbrace{\begin{bmatrix} y(k) & \cdots & y(1) \end{bmatrix}}_{Y(k,1)} - \hat{\theta} \otimes \underbrace{\begin{bmatrix} p(k) & \cdots & p(1) \end{bmatrix}}_{P(k,1)}$$

or

$$\Xi(k,1) = Y(k,1) - \hat{\theta} \otimes P(k,1)$$

As shown in [6] the solution that minimizes the prediction error $\Xi(k,1)$ corresponds to the greatest solution of the inequality

$$Y(k,1) \geq \hat{\theta} \otimes P(k,1)$$

and can be computed using the min–plus algebraic operators "$\oplus'$" and "$\otimes'$":

$$\begin{aligned} \hat{\theta}_i &= \bigoplus_{j=1}^{k}{}' \, Y_j(k,1) \otimes' \big(- P_{ij}(k,1) \big) \\ &= \min_{j=1,\ldots,k} \big(y(j) - p_i(j) \big) \end{aligned} \tag{3}$$

where $Y_j(k,1)$ denotes the j-th column of $Y(k,1)$. For this solution, the following properties hold [9]:

$$\hat{\theta}_i \geq \theta_i \tag{4}$$

$$\hat{\theta} \otimes P(k) = \theta \otimes P(k) \tag{5}$$

such that the prediction error $\xi(j) = 0$, for $j = 1, \ldots, k$ due to (5). On the other hand, property (4) shows that in general, the parameters will be overestimated by this approach. In [12] this issue has been investigated and a condition was given for convergence of the estimated parameters to their true values.

Hence, an initial estimate for the system parameters can be obtained based on k data points using (3). To track changing system parameters, an update of the estimates after each update of the output is necessary. A first possibility is the recursive evaluation of (3) as first proposed in [10] for the estimation of the system's impulse response. Thus,

$$\begin{aligned} \hat{\theta}_i(k) &= \bigoplus_{j=1}^{k}{}' \big(y(j) - p_i(j) \big) \\ &= \bigoplus_{j=1}^{k-1}{}' \big(y(j) - p_i(j) \big) \oplus' \big(y(k) - p_i(k) \big) \\ &= \hat{\theta}_i(k-1) \oplus' \big(y(k) - p_i(k) \big) \\ &= \min \Big(\hat{\theta}_i(k-1), \big(y(k) - p_i(k) \big) \Big) \end{aligned}$$

However, since $\oplus'$ corresponds to minimization, an update where $\big(y(k) - p_i(k)\big) > \hat{\theta}_i(k-1)$ will not have any influence on $\hat{\theta}_i(k)$. Thus, increasing parameter values will not be detected by this approach. As a possible solution to this problem the estimation can be carried out considering only the most recent N_e data points, and choosing

$$\underbrace{\begin{bmatrix} \xi(k) & \cdots & \xi(k-N_e) \end{bmatrix}}_{\Xi(k,k-N_e)} = \underbrace{\begin{bmatrix} y(k) & \cdots & y(k-N_e) \end{bmatrix}}_{Y(k,k-N_e)} - \hat{\theta} \otimes \underbrace{\begin{bmatrix} p(k) & \cdots & p(k-N_e) \end{bmatrix}}_{P(k,k-N_e)}$$

However, using the reasoning above, it can be concluded that a change in a parameter θ_i that leads to measurements with $y(j) - p_i(j) > \hat{\theta}_i(k)$ may be detected only when all N_e data points considered in the estimation are influenced by this new parameter value.

Therefore, the algorithm used in the sequel is based on a different strategy. Assume, that the initial estimation $\hat{\theta}(0)$ was determined from the first N_e data points by (3). Similar to the conventional recursive estimation algorithms, the new estimate can be computed by adding a weighted difference between the new measurement and the measurement predicted by the model. This principle was used in [9] (though the similarity to the conventional recursive estimation was not pointed out) and will be applied for adaptive MPC with some modifications. Let $\hat{\theta}(k-1)$ be the estimate at the end of the $(k-1)$th cycle. If $\hat{\theta}(k-1)$ satisfies $y(k) = \hat{\theta}(k-1) \otimes p(k)$, we choose

$\hat{\theta}(k) = \hat{\theta}(k-1)$. If not, then $\hat{\theta}(k)$ is obtained by the series

$$\begin{cases} \hat{\theta}^{(0)}(k) &= \hat{\theta}(k-1) \\ \hat{\theta}^{(\ell)}(k) &= \hat{\theta}^{(\ell-1)}(k) + \alpha \Delta^{(\ell-1)}(k) \quad \ell > 0 \end{cases} \tag{6}$$

where $0 < \alpha \leq 2$ is a weighting parameter and

$$\Delta^{(\ell-1)}(k) = \Big[\Big(Y(k, k-N_{\mathrm{e}}) \otimes' \big(-P^T(k, k-N_{\mathrm{e}})\big)\Big) - \Big(\big(\hat{\theta}^{(l-1)(k)} \otimes P(k, k-N_{\mathrm{e}})\big) \otimes' \big(-P^T(k, k-N_{\mathrm{e}})\big)\Big)\Big] \tag{7}$$

In [9] it is proven that for $\alpha = 1$ the iteration (6)-(7) will converge to a value that satisfies $y(k) = \hat{\theta}^{(\ell)}(k) \otimes p(k)$.

Note that in contrast to [9], in this paper we use an MPLIO model rather than an impulse response model. The MPLIO description is more compact and so the estimation can be done using less information. Furthermore we have two new parameters: N_{e}, the number of past values of input and outputs, and the parameter α, which can be used to tune the convergence rate of the recursive estimation algorithm.

4 Model predictive control for MPLIO systems

In [5] we have extended the MPC framework to MPL state-space models. Following the strategy for conventional discrete-time systems in an input-output setting [3] we define a cost criterion $J(k)$ that reflects the due-date error and input-buffer cost in the event period $[k, k+N_{\mathrm{p}}-1]$:

$$J(k) = \sum_{j=0}^{N_{\mathrm{p}}-1} \Big(\hat{y}(k+j|k) - r(k+j)\Big) - \lambda u(k+j) \tag{8}$$

where N_{p} is the prediction horizon and λ is a weighting parameter, $\hat{y}(k+j|k)$ is the prediction of the output signal $y(k+j)$, based on the knowledge at event step k and $r(k)$ is the due date signal. Other choices for cost function J are given in [5]. In order to compute the optimal MPC input signal, we need to make predictions of the output signal.

Lemma 1 *Consider an MPLIO system (1)-(2). For any non-negative integer j, there exist polynomials*

$$C_j(\gamma) = c_{1,j} \otimes \gamma^1 \oplus \ldots \oplus c_{n,j} \otimes \gamma^n \tag{9}$$

$$D_j(\gamma) = d_{0,j} \otimes \gamma^0 \oplus \ldots \oplus d_{m-1,j} \otimes \gamma^{m-1} \tag{10}$$

$$F_j(\gamma) = f_{0,j} \otimes \gamma^0 \oplus \ldots \oplus f_{j,j} \otimes \gamma^j \tag{11}$$

such that

$$\hat{y}(k+j|k) = C_j(\gamma)y(k) \oplus D_j(\gamma)u(k-1) \oplus F_j(\gamma)u(k+j) \tag{12}$$

The proof is in [13].

Note that in (12) the first part of the expression, $C_j(\gamma)y(k-1) \oplus D_j(\gamma)u(k-1)$, only depends on values of previous event steps and the second part of the expression, $F_j(\gamma)u(k+i)$, only on present and future values of the input signal.

Using the results of lemma 1, we can construct matrices that relate the future output signal with past values of the output and future values of the input. By defining the vector

$$\tilde{y}_0(k) = \begin{bmatrix} C_0(\gamma)y(k) \oplus D_0(\gamma)u(k-1) \\ \vdots \\ C_{N_{\mathrm{p}}-1}(\gamma)y(k) \oplus D_{N_{\mathrm{p}}-1}(\gamma)u(k-1) \end{bmatrix},$$

and the constant matrix

$$\tilde{F} = \begin{bmatrix} f_{0,0} & \varepsilon & \cdots & \varepsilon \\ f_{0,1} & f_{1,1} & & \vdots \\ \vdots & & \ddots & \\ f_{0,N_{\mathrm{p}}-1} & \cdots & & f_{N_{\mathrm{p}}-1,N_{\mathrm{p}}-1} \end{bmatrix},$$

we obtain $\tilde{y}(k) = \tilde{y}_0(k) \oplus \tilde{F} \otimes \tilde{u}(k)$, where

$$\tilde{y}(k) = \begin{bmatrix} \hat{y}(k|k) \\ \vdots \\ \hat{y}(k+N_{\mathrm{p}}-1|k) \end{bmatrix}, \tilde{u}(k) = \begin{bmatrix} u(k) \\ \vdots \\ u(k+N_{\mathrm{p}}-1) \end{bmatrix}$$

The aim is now to compute an optimal input sequence $\tilde{u}(k)$ that minimizes $J(k)$ subject to constraints on the inputs and outputs. These constraints are due to limits on the input and output event separation times or due to maximum due dates for the output events. Since the elements of $u(k)$ correspond to consecutive event occurrence times, we have the additional condition $\Delta u(k+j) = u(k+j) - u(k+j-1) \geq 0$ for $j = 0, \ldots, N_{\mathrm{p}}-1$. Furthermore, in order to reduce the number of decision variables and the corresponding computational complexity we introduce a control horizon N_{c} ($\leq N_{\mathrm{p}}$) and we impose the additional condition that the input rateshould be constant from event step $k + N_{\mathrm{c}} - 1$ on: $\Delta u(k+j) = \Delta u(k+N_{\mathrm{c}}-1)$ for $j = N_{\mathrm{c}}, \ldots, N_{\mathrm{p}}-1$, or equivalently $\Delta^2 u(k+j) = \Delta u(k+j) - \Delta u(k+j-1) = 0$ for $j = N_{\mathrm{c}}, \ldots, N_{\mathrm{p}} - 1$.

MPC uses a receding horizon principle. This means that after computation of the optimal control sequence $u(k), \ldots, u(k + N_{\mathrm{c}} - 1)$, only the first control sample $u(k)$ will be implemented, subsequently the horizon is shifted one event step, and the optimization is restarted with new information of the measurements. The MPC problem for MPL systems for event step k is formulated as follows (compare with [5] for the state space case):

$$\min_{\tilde{u}(k),\tilde{y}(k)} J(\tilde{u}(k), \tilde{y}(k)) \tag{13}$$

subject to

$$\tilde{y}(k) = \tilde{y}_0(k) \oplus \tilde{F} \otimes \tilde{u}(k) \tag{14}$$

$$A_{\mathrm{c}}(k)\tilde{u}(k) + B_{\mathrm{c}}(k)\tilde{y}(k) \leqslant c_{\mathrm{c}}(k) \tag{15}$$

$$\Delta u(k+j) \geqslant 0 \quad \text{for } j=0, \ldots, N_{\mathrm{p}}-1 \tag{16}$$

$$\Delta^2 u(k+j) = 0 \quad \text{for } j=N_{\mathrm{c}}, \ldots, N_{\mathrm{p}}-1, \tag{17}$$

where equation (15) reflects the constraints on the inputs and outputs. Similar to [5] we can prove that, if the linear constraints are monotonically non-decreasing as a function of $\tilde{y}(k)$, the MPL-MPC problem can be recast as a convex problem. Moreover, by introducing some additional dummy variables the problem can even be reduced to a linear programming problem (see [5]).

5 The adaptive MPC algorithm

The two important ingredients of the adaptive controller, identification and control law, have been discussed in the previous sections. This leads to the final adaptive MPC algorithm, which consists of the following 5 steps.

step 1 (initial identification): The model is initialized by computing $\hat{\theta}_0$ using equation (3).

step 2 (measurement): Obtain new measurement $y(k)$ at event step k.

step 3 (adaptation): Make a recursive estimation of $\hat{\theta}^{(k)}$ using equation (6)-(7).

step 4 (control law): Compute new control sequence $\tilde{u}^*(k)$ by solving the MPL-MPC problem, which is defined by the optimization of (13) subject to (14)-(17). The first element $u(k)$ of $\tilde{u}^*(k)$ is fed to the system.

step 5 (receding horizon): The horizon is shifted one step. Return to step 2.

6 Example

Consider the MPLIO system, described by the input-output relation (1) where $A(\gamma)$ and $B(\gamma)$ are polynomial operators

$$\begin{aligned} A(\gamma) &= a_1 \otimes \gamma^1 \oplus a_2 \otimes \gamma^2 \quad , \\ B(\gamma) &= b_0 \otimes \gamma^0 \oplus b_1 \otimes \gamma^1 \oplus b_2 \otimes \gamma^2 \end{aligned}$$

Define the parameter vector

$$\theta = \begin{bmatrix} a_1 & a_2 & b_0 & b_1 & b_2 \end{bmatrix}$$

We simulate the system for $k = 1, \ldots, 300$ where

$$\theta = \begin{cases} [\, 0.2\ 0.4\ 0.2\ 0.4\ 0.6 \,] & , 1 \leq k \leq 100 \\ [\, 0.25\ 0.7\ 0.3\ 0.55\ 1.0 \,] & , 101 \leq k \leq 200 \\ [\, 0.3\ 0.6\ 0.3\ 0.6\ 0.9 \,] & , 201 \leq k \leq 300 \end{cases}$$

An adaptive model predictive controller strategy is applied following section 5. The due date signal $r(k)$ is a non-decreasing random[3] signal with an average slope of 0.4175 and variance 0.3470. The initial state is set to $p(0) = [\, 0\ 0\ 0\ 0\ 0 \,]^T$ and the criterion function is given by (13) for $N_{\mathrm{p}} = 10$, $N_{\mathrm{c}} = 2$ and $\lambda = 0.01$. For each k, the model is updated using an update interval with $N_{\mathrm{e}} = 15$ and $\alpha = 1$, and (with the updated model) the optimal input sequence is computed, and finally the first element $u(k)$ of the sequence $\tilde{u}(k)$ is applied to the system (due to the receding horizon strategy). Figure 1 gives the due date error, i.e. the difference between the due date signal and the output signal $y(k)$. Note that only near the jumps of the parameters the due date error is positive, corresponding to a due date violation. Figure 2 shows the model parameters, as estimated by the identification algorithm. Note that in the first interval ($k = 1, \ldots, 100$), the second parameter is not estimated accurately, and in the third interval ($k = 201, \ldots, 300$) the fourth parameter has a deviation. Clearly, these parameters are not important in the prediction of the future behaviour, because the predictive control algorithm results in a good due date tracking behaviour.

[3]The due date signal is chosen random to express the varying customer demand.

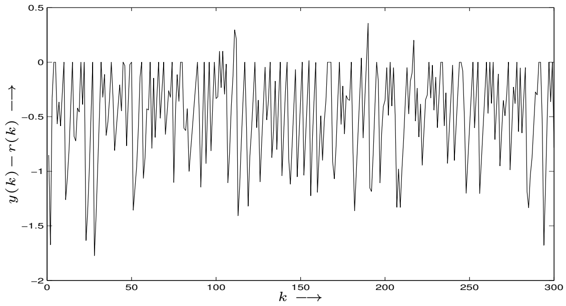

Figure 1: Due date error $y(k)-r(k)$

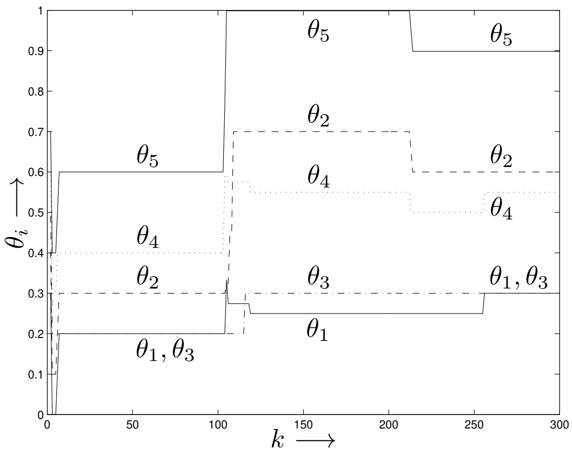

Figure 2: Estimated parameters $\hat{\theta}$

7 Discussion

In this paper we have derived a technique for adaptive MPC of MPL systems, given an input-output description. We have included the identification and estimation update into the algorithm. If the linear constraints are a non-decreasing function of the output the computation of the MPC control law can be done using a linear programming algorithm. An simulation example has shown that the algorithm gives a good closed-loop behaviour in the case of a MPLIO model with time-varying parameters.

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